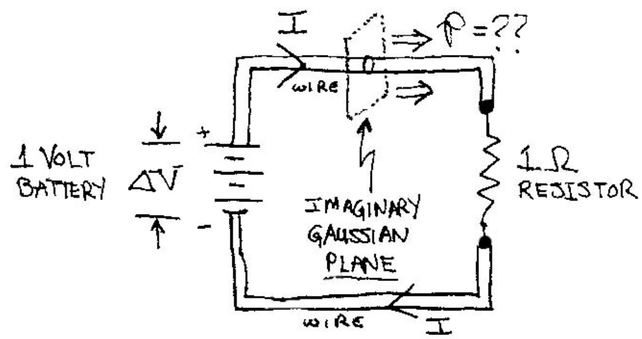
EM Power *Transport* Down / Along a Long Wire Carrying a Steady / DC Current *I*

Consider a steady/DC current of I = 1.0 Amperes flowing in the circuit shown in the figure below:



The battery/power supply voltage is $\Delta V = 1.0$ volts. Since the total resistance of the copper wire is $\ll$ less than that of the $1.0\,\Omega$ resistor {*e.g.* 20 AWG pure copper wire has a diameter of D = 0.032" (~ 1/32nd inch) and has a resistance of $10.150\,\Omega$ per 1000 ft (@ T = 20 º C) – see/refer to the Table of American Wire Gauge Wire Sizes in Appendix A at the end of these lecture notes}, thus 1 *m* (~ 3 *ft*) of 20 AWG pure copper wire has a resistance of ~ $0.033\,\Omega$ {@ T = 20ºC}) so {temporarily} we will neglect the resistance of the copper wire. Thus, with this simplifying assumption, the potential difference across the resistor is also $\Delta V = 1.0$ volts.

From Ohm's law $\{\Delta V = IR\}$ the steady/DC current flowing through this circuit is $I = \Delta V/R = 1.0$ Amperes and thus the power dissipated in the $1.0\,\Omega$ resistor is:

$$P_{resistor} = \Delta V * I = I^2 R = (1.0\text{ Amp})^2 \times 1.0\,\Omega = 1.0\text{ Watts}\,.$$

The electrical power is supplied by the battery {the chemical potential energy in the battery is transformed into electrical energy} and is dissipated as heat in the resistor due to Joule heating associated with {the ensemble of} conduction electrons scattering off of atoms in the resistor. Obviously, electrical {*i.e. EM*} power has to be transported from the battery to the resistor. We ask: precisely how does this occur?

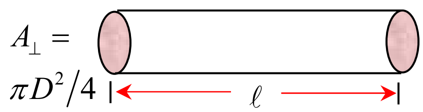
As previously discussed (last semester, in P435 Lect. Notes 21) we learned that the mechanical power associated with the kinetic energy of (free/conduction/drift) electrons flowing in the wire (*e.g.* made of pure copper wire, mean/avg. electron drift velocity $\langle v_D\rangle$ = 75 μm/sec) cannot account for the P = 1.0 Watt power transported down/along a wire, where it is dissipated as heat (*i.e.* thermal energy) in the $1.0\,\Omega$ resistor:

$$\boxed{P^{e^-}_{\substack{cu\\mech}} = \left(\frac{I}{q_e}\right)\frac{1}{2}m_e\langle v_D\rangle^2 \simeq \frac{1.0\,Amp}{1.6\times10^{-19}\,Coul}\left(\frac{1}{2}\right)9.1\times10^{-31}kg*\left(75\times10^{-6}\,\text{m/s}\right)^2}$$

conduction electrons/sec crossing imaginary Gaussian plane

$$\boxed{P^{e^-}_{\substack{cu\\mech}} \simeq 1.6\times10^{-20}\text{ Watts }\left(\text{Joule}/\text{sec}\right) \ll 1\text{ Watt}}$$

Now, the copper wires connecting the 1.0 V battery to the 1.0Ω resistor do indeed have finite resistance, R_{wire} and thus the wires will also dissipate some electrical power. The resistivity of (pure) copper (@ $T = 20^o$ C) is $\rho_{cu}\left(T = 20^o C\right) = 1.68\times10^{-8}\,\Omega-$meters. The resistance of copper wire, in terms of its resistivity ρ_{cu}, length ℓ and cross-sectional area $A_\perp$ is: $\boxed{R_{\substack{cu\\wire}} = \rho_{cu}\ell/A_\perp}$



Referring to the Table of American Wire Gauge Wire Sizes (Appendix A of these lecture notes) we see that pure copper wire of 20 AWG has a diameter D of D (20 AWG) = 0.0320" (= 0.8128 mm) and has a resistance of 10.150 Ω/1000 ft. (= 33.292 Ω/km = $33.292\times10^{-3}\,\Omega/\text{m}$) $\left\{@T = 20^o C\right\}$.

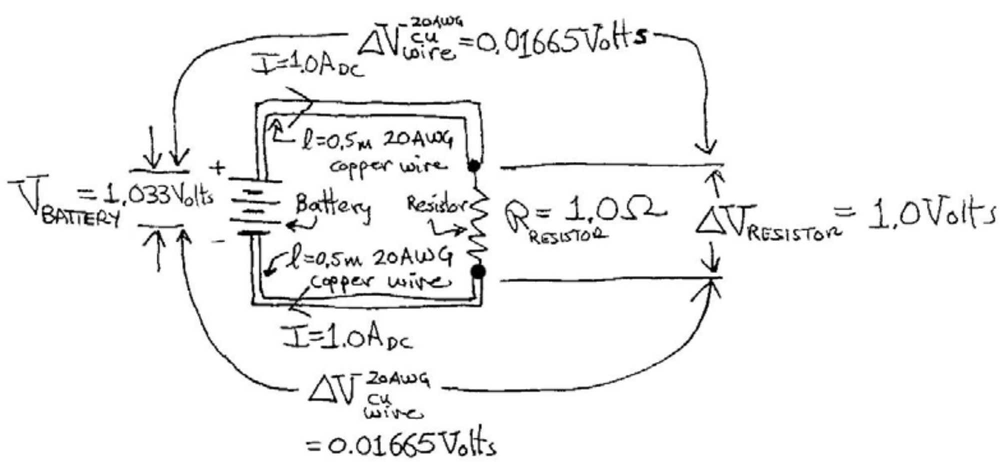
Thus, two 20 AWG copper wires (each with wire length $\ell = 0.5\,m$) connecting battery to the resistor and carrying a steady/DC current of I = 1.0 Amperes, have a resistance of:

$$\boxed{R^{20AWG}_{\substack{cu\\wire}} = 33.292\times10^{-3}\,\Omega/m\times\left(\ell = 0.5\,m\right) = 0.01665\,\Omega} \qquad \text{(for each wire)}$$

The DC voltage drop across each 0.5 m long wire lead is thus (for I = 1.0 Amp):

$$\boxed{\Delta V^{20AWG}_{\substack{cu\\wire}} = I * R^{20AWG}_{\substack{cu\\wire}} = 1.0\,A * 0.01665\ \Omega = 0.01665\ \text{Volts}} \ \text{(for each wire)}$$

Thus, in reality the battery would actually have to supply a DC voltage of $\Delta V_{batt} = 2*0.01665\,\text{V} + 1.0\,\text{V} = 1.0333$ Volts in order to provide a DC current of I = 1.0 Amps flowing through this circuit:



Electrical/*EM* power dissipated in circuit = Joule heating of wires and resistor (i.e. *EM* energy ultimately winds up as heat / thermal energy

The *EM* power dissipated in each ℓ = 0.5 m long 20 AWG copper wire is:

$$P_{\substack{\text{20AWG}\\\text{cu wire}}} = \Delta V^{20AWG}_{\substack{cu\\wire}} * I = I^2 R^{20AWG}_{\substack{cu\\wire}} = \left(1.0\text{A}\right)^2 * 0.01665\ \Omega = 0.01665\ \text{Watts}$$

{n.b.: $P_{\substack{\text{20AWG}\\\text{cu wire}}} = 0.01665\,\text{Watts} = 16.65\ \text{milliwatts}\ \text{(mW)} << P_{resistor} = 1.0\ \text{Watts}$ }

The total *EM* power dissipated in the circuit = $2*P_{\substack{\text{20AWG}\\\text{cu wire}}} + P_{resistor} = 1.0333$ Watts

$$P_{TOT} = \Delta V_{TOT} * I = 1.0333\ \text{Volts} * 1.0\ \text{Amps} = 1.0333\ \text{Watts}$$
$$= P_{battery} = \text{power supplied by 1.0333 Volt battery.}$$

So how *is* the electrical/*EM* power transported {by the copper wires} from the power source (battery) to the resistor???

The "short" answer is that the *EM* power is transported from the battery to the resistor by the electromagnetic field(s) associated with the steady current *I* flowing through the circuit, in proximity to the wire. There exists *another* Poynting's Vector (which we have not yet discussed, but are about to…) that *is* responsible for the bulk of the *EM* power transport – which points in the direction of (conventional) current flow, precisely as one would expect!!!

The "long" answer is as follows:

This discussion (again) is a tale involving two (inertial) reference frames – thus, an astute reader should instantly realize that (special) relativity *is* intimately involved here, however the relative speeds of the electric charge carriers involved are all "glacial", *i.e.* $v \ll c$ and thus we do not need to explicitly "haul in" all of the mathematical formalism/mathematic rigor of (special) relativity in order to understand the physics {we are consciously avoiding doing this, especially since we have not yet discussed special relativity and relativistic electrodynamics in this course – which we will be doing so before the end of the semester – and thus we will return to this same problem at the appropriate moment and discuss it again from a relativistic point of view at that time…}.

In the discussion here, we also specifically point out an important charge-asymmetry aspect of matter at the microscopic level – namely that of negative-charged, very light "gas" of "free" conduction electrons in a metal wire vs. positively charged, heavy atoms that are bound together in a rigid/fixed three-dimensional lattice that makes up the macroscopic metal conducting wire. Because of the fact that a (real) wire has this two-component aspect associated with it at the microscopic level, the superposition principle is also needed in order to clearly understand the physics.

The nature of the discussion here is also consciously/deliberately simplified in order to avoid getting bogged down in the complexities associated with (even simple) electrical circuits in the real world; nevertheless the simplified discussion will serve its intended purpose, that of highlighting the salient physics that is involved.

Note that one important tacit/implicit assumption is made regarding the physics associated with a steady/DC electric current flowing in a conducting wire – namely that the conducting wire remains (overall) electrically neutral – i.e. there is no *net* electric charge present on the wire when an electric current flows through it. This makes intuitive sense, since an initially uncharged electrically conducting wire at the microscopic level consists of equal numbers of a "gas" of negatively-charged "free" electrons and positive-charged atoms, the latter of which are bound together in a rigid/fixed three-dimensional lattice {which comprises macroscopic conducting metal wire}. Note also that an electrically neutral wire is also the lowest possible energy configuration of that wire – any build-up of a net electric charge on the wire is costly, in terms of requiring an additional input of energy to do so!

When a steady/DC current I flows *e.g.* in a (long) 20 AWG pure copper wire, if the wire is at rest in the lab frame, the (assumed uniform) longitudinal electric field that is set up inside the long wire causes the "gas" of "free" conduction electrons to flow down the wire with (assumed) constant/uniform drift velocity $\vec{v}_{e^-} = \langle \vec{v}_D \rangle$ (*n.b.* the direction that these drift electrons move is opposite to that of conventional current). This "gas" of 'free' conduction electrons thus "percolates" as a "coherent" wind flowing through the three-dimensional, cylindrically-shaped, fixed/rigid lattice of positive-charged atoms (*e.g.* copper atoms, for a copper wire). Note that the "free" conduction electrons are in motion in the lab reference frame, whereas the positive-charged atoms bound together in a 3-D lattice are at rest in the lab frame.

In the lab frame, the "free" conduction electrons flowing in the 20 AWG pure copper wire have (glacial) $\vec{v}_{e^-} = \langle \vec{v}_D \rangle = -v_D \hat{z} \simeq -75 \text{ mm/sec}\,\hat{z}$ drift velocity, flowing / moving in the $-\hat{z}$ direction. For simplicity's sake, we will also ignore/neglect the (Maxwell-Boltzman type) thermal fluctuations in the 3-D velocity distribution of electron drift velocities (we tacitly assume that the wire is e.g. at room temperature ($T = 20^o$C), and simply assume that each conduction electron has the same/identical lab velocity vector $\vec{v}_{e^-} = \langle \vec{v}_D \rangle \simeq -75 \text{ mm/sec}\,\hat{z} = 75 \text{ mm/sec}(-\hat{z})$ in the $-\hat{z}$ direction (only). Thus, the "free"/conduction electron "gas" flows through the 3-D lattice / matrix of copper atoms as a "coherent" wind, driven by the longitudinal $\vec{E}$-field:

$$\boxed{\vec{E}^{wire}_{Longitudinal} = \frac{\vec{J}_{free}}{\sigma_C} = \frac{\Delta V^{20\text{AWG}}_{\text{Cu wire}}}{\ell}\hat{z} = \frac{0.01665 \text{ Volts}}{0.5 \text{ m}}\hat{z} = 0.0333 \text{ Volts/m}\,\hat{z}}$$

n.b. $\vec{J}_{free}, \sigma_C$ and hence $\vec{E}^{20\text{AWG}}_{\text{Cu wire}}$ assumed to be uniform

In the lab reference frame (where the wire is at rest), since the "free" conduction electrons are in uniform motion, an observer at rest in the lab frame see no net electric field because the current-carrying wire is overall electrically neutral – the static, radial electric fields associated with the uniform/constant volume electric charge densities $\rho_+^{Cu\ atoms}(\vec{r}\,') = const.$ and $\rho_-^{e^-}(\vec{r}\,') = -const. = -\rho_+^{Cu\ atoms}(\vec{r}\,')$ cancel each other {*n.b.* these two electric fields in the rest frame of the conducting wire must be radial from symmetry considerations associated with the cylindrical geometry of the wire}:

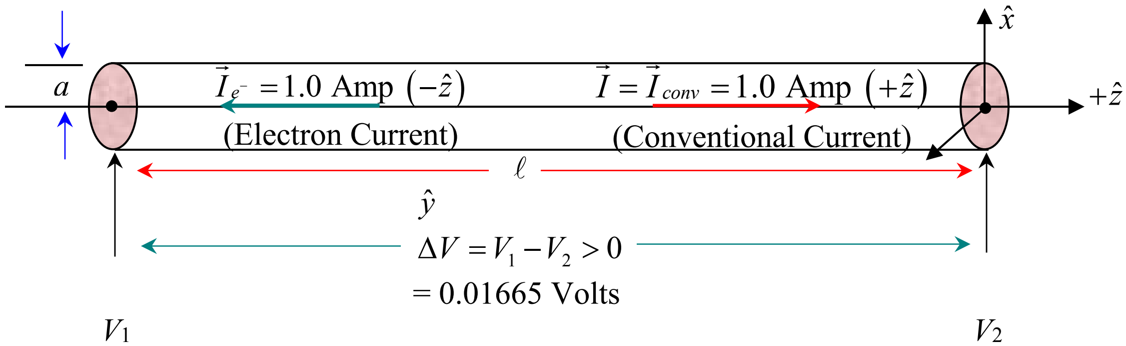
$$\boxed{\vec{E}^{tot}_{Lab}(\rho) = \vec{E}_+^{Cu\ atoms}(\rho) + \vec{E}_-^{e^-}(\rho) = \frac{1}{4\pi\varepsilon_o}\int_{v'} \frac{\rho_+^{Cu\ atoms}(\vec{r}\,')}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau' + \frac{1}{4\pi\varepsilon_o}\int_{v'} \frac{\rho_-^{e^-}(\vec{r}\,')}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau' = 0}$$

However, an observer in the rest frame of the conducting wire also sees a magnetic field, which arises from the (assumed) uniform collective motion of the "free" conduction electrons at the microscopic level, corresponding to a macroscopic uniform electron current density $\vec{J}_{e-} = n_{e-} e \vec{v}_{e-} = -\vec{J}_{conv} = -J\hat{z}$, with $\vec{v}_{e^-} = \langle \vec{v}_D \rangle = -\langle v_D \rangle \hat{z}$ and a macroscopic uniform electron current $\vec{I}_{e-} = -\vec{I}_{conv} = -I\hat{z} = -J_{e-} A_\perp^{wire}\hat{z}$. Using Ampere's circuital law, the B-field observed in the rest frame of the wire is:

$$\boxed{\vec{B}^{inside}(\rho \le a)\Big|_{lab\ frame} = +\left(\frac{\mu_o I \rho}{2\pi a^2}\right)\hat{\varphi}} \text{ and } \boxed{\vec{B}^{outside}(\rho \ge a)\Big|_{lab\ frame} = +\left(\frac{\mu_o I}{2\pi \rho}\right)\hat{\varphi}} \text{ (SI units: Tesla)}$$

$\rho = \sqrt{x^2 + y^2}$ in cylindrical coordinates, $a = \frac{1}{2}D(20 \text{ AWG})$

Lab Reference Frame (wire at rest):



Let us now go into the rest frame of the "free" / conduction electron gas / "wind" flowing through the 3-D lattice / matrix of copper atoms in the 20 AWG wire. In this reference frame, since the electrons are at rest, thus there is no magnetic field {as collectively generated by the electrons} because the electron current $I_{e^-} = 0$ in the rest frame of the electrons! Ampere's law: $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{encl}^{e^-} = 0$ because $I_{encl}^{e^-} = 0$!!! However, an observer in the rest frame of the "free"/ conduction electrons sees the 3-D lattice / matrix of positive-charged copper atoms coherently / en mass moving with uniform velocity of $\vec{v}_{Cu\ atom}\big|_{e^- rest\ frame} = -\vec{v}_e\big|_{labframe} = 75\ \text{mm/sec}\left(+\hat{z}\right)$ in the $+\hat{z}$ direction. These copper atoms have charge, $Q_{cu} = +1e$ (since copper metal has one conduction electron / copper atom), and thus, in the rest frame of the "free" / conduction electron "gas", a magnetic field is indeed present; again from Ampere's Law: $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{encl}$ where I_{encl} = the macroscopic steady/DC current associated with flow of positive charged copper atoms in / as seen by an observer in the rest frame of the electron "gas" in the copper metal of 20 AWG wire:

$$\boxed{\vec{I}_{Cu\ atoms}\Big|_{e^- rest\ frame} = -\vec{I}_{e^-}\Big|_{lab\ frame} = -\left(-\vec{I}_{conv}\right) = +\vec{I}_{conv}\Big|_{lab\ frame} = 1.0\ \text{Amp}\left(+\hat{z}\right)}$$

Thus: $\boxed{\vec{B}^{inside}_{Cu\ atoms}\left(\rho \le a\right)\Big|_{\substack{e^- rest \\ frame}} = +\left(\dfrac{\mu_o I \rho}{2\pi a^2}\right)\hat{\varphi}}$ and: $\boxed{\vec{B}^{outside}_{Cu\ atoms}\left(\rho \ge a\right)\Big|_{\substack{e^- rest \\ frame}} = +\left(\dfrac{\mu_o I}{2\pi \rho}\right)\hat{\varphi}}$

Thus, an observer at rest in the lab frame (where the wire is at rest) sees the same magnetic field as an observer in the rest frame of the "gas" of drift/conduction electrons, however why/how these magnetic fields arise in each reference frame is associated with the charge species in motion in each of the two reference frames! The drift/conduction electrons are in motion in the lab frame and create the B-field observed in that reference frame, whereas the positive-charged atoms are in motion (in the opposite direction) in the rest frame of the "gas" of drift/conduction electrons and create the B-field observed in that reference frame!

Let us now digress here for a moment, and discuss the academic/ivory-tower problem of the motion of an isolated electric charge q moving with constant/uniform velocity $\vec{v}_{lab}$ in the lab reference frame, in which in an external/applied magnetic field $\vec{B}^{ext}_{lab}$ is present. As we have discussed in previous lectures - last semester in P435 - the moving electrically-charged particle will experience a (magnetic) Lorentz force acting on it:

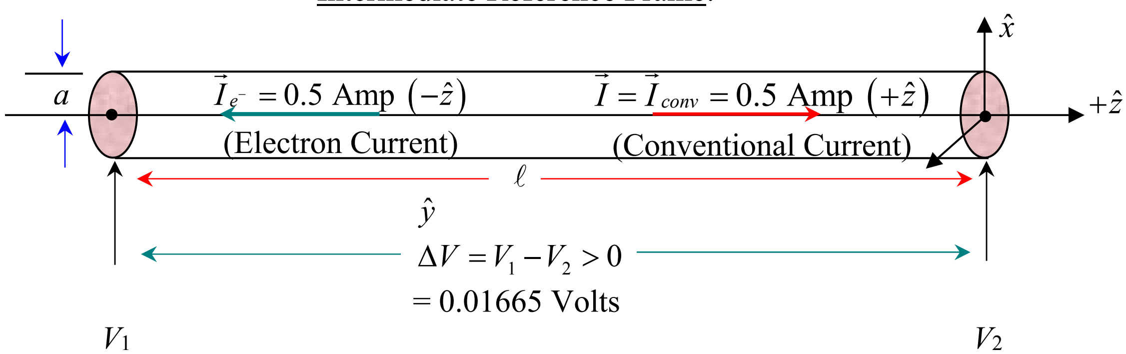
$$\boxed{\vec{F}_{lab}^{mag}(\vec{r}) = q\vec{v}_{lab}(\vec{r})\times\vec{B}_{lab}^{ext}(\vec{r})}$$

Note also that in this formula, the fact that the electrically-charged particle moving with velocity $\vec{v}_{lab}$ also generates its <u>own</u> solenoidal magnetic field is of no consequence/has no impact on affecting and/or altering this force acting on the electrically-charged particle itself.

In the <u>rest frame</u> of this isolated, electrically charged particle, again as we have discussed in previous lectures (in last semester's P435 course) an observer sees an electric field of $\boxed{\vec{E}_{rest}(\vec{r}') = \vec{v}_{lab}(\vec{r}')\times\vec{B}_{lab}^{ext}(\vec{r}')}$ and a corresponding (electric) force in the rest frame of the electrically-charged particle of $\boxed{\vec{F}_{rest}^{elect}(\vec{r}') = q\vec{E}_{rest}(\vec{r}') = q\vec{v}_{lab}(\vec{r}')\times\vec{B}_{lab}^{ext}(\vec{r}')}$.

Now let us go back to our original current-carrying wire problem, but <u>instead</u> (to avoid some conceptual difficulties) we will go into a <u>third</u>, "intermediate" reference frame which "splits the difference" between the <u>lab</u> frame (where the copper atoms are at rest and the "free"/conduction electrons are in motion) and the <u>rest</u> frame of the "gas" of "free"/conduction electrons (where the "free/conduction electrons are at rest and the copper atoms are in motion). This "intermediate" reference frame is where the "gas" of "free"/conduction electrons is moving to the <u>left</u> with (assumed) constant/uniform velocity $\boxed{\vec{v}_{e^-} = -\tfrac{1}{2}\langle v_D\rangle\hat{z}}$ and the copper atoms are moving to the <u>right</u> with (assumed) constant/uniform velocity $\boxed{\vec{v}_{Cu\ atoms} = +\tfrac{1}{2}\langle v_D\rangle\hat{z} = -\vec{v}_{e^-}}$, as shown in the figure below:

<u>Intermediate Reference Frame</u>:



In this intermediate reference frame, because $\boxed{\vec{v}_{e^-} = -\tfrac{1}{2}\langle v_D\rangle\hat{z}}$ and $\boxed{\vec{v}_{Cu\ atoms} = +\tfrac{1}{2}\langle v_D\rangle\hat{z} = -\vec{v}_{e^-}}$ then:

$$\boxed{\vec{J}_{e-} = n_{e-}e\vec{v}_{e-} = -\tfrac{1}{2}n_{e-}e\langle v_D\rangle\hat{z} = -\tfrac{1}{2}\vec{J}_{conv} = -\tfrac{1}{2}J\hat{z}} \quad \text{and}$$

$$\boxed{\vec{J}_{Cu\ atoms} = n_{Cu\ atoms}e\vec{v}_{Cu\ atoms} = +\tfrac{1}{2}n_{Cu\ atoms}e\langle v_D\rangle\hat{z} = +\tfrac{1}{2}\vec{J}_{conv} = +\tfrac{1}{2}J\hat{z}}$$

and thus:

$$\boxed{\vec{I}_{e-} = -J_{e-}A_\perp^{wire}\hat{z} = -\tfrac{1}{2}J_{conv}A_\perp^{wire}\hat{z} = -\tfrac{1}{2}\vec{I}_{conv} = -\tfrac{1}{2}I\hat{z}} \text{ and}$$

$$\boxed{\vec{I}_{Cu\ atoms} = +J_{Cu\ atoms}A_\perp^{wire}\hat{z} = +\tfrac{1}{2}J_{conv}A_\perp^{wire}\hat{z} = +\tfrac{1}{2}\vec{I}_{conv} = +\tfrac{1}{2}I\hat{z}}$$

and thus we also see in this intermediate reference frame that there are two separate contributions to the overall/total magnetic field present in this reference frame:

$$\boxed{\vec{B}_{e^-}^{inside}(\rho\le a)\Big|_{\substack{intermed\\ ref\ frame}} = +\left(\frac{\mu_o I\rho}{2\pi a^2}\right)\hat{\varphi}} \quad \text{and} \quad \boxed{\vec{B}_{e^-}^{outside}(\rho\ge a)\Big|_{\substack{intermed\\ ref\ frame}} = +\frac{1}{2}\left(\frac{\mu_o I}{2\pi\rho}\right)\hat{\varphi}}$$

$\boxed{\vec{B}^{inside}_{Cu\ atoms}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} = +\left(\frac{\mu_o I\rho}{2\pi a^2}\right)\hat{\varphi}}$ and $\boxed{\vec{B}^{outside}_{Cu\ atoms}(\rho \ge a)\Big|_{\substack{intermed\\ ref\ frame}} = +\frac{1}{2}\left(\frac{\mu_o I}{2\pi\rho}\right)\hat{\varphi}}$

The total magnetic field observed in this intermediate reference frame is:

$$\boxed{\vec{B}_{tot}(\rho)\Big|_{\substack{intermed\\ ref\ frame}} = \vec{B}_{e^-}(\rho)\Big|_{\substack{intermed\\ ref\ frame}} + \vec{B}_{Cu\ atoms}(\rho)\Big|_{\substack{intermed\\ ref\ frame}}}$$

thus:

$$\boxed{\vec{B}^{inside}_{tot}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} = \vec{B}^{inside}_{e^-}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} + \vec{B}^{inside}_{Cu\ atoms}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} = +\left(\frac{\mu_o I\rho}{2\pi a^2}\right)\hat{\varphi}}$$

$$\boxed{\vec{B}^{outside}_{tot}(\rho \ge a)\Big|_{\substack{intermed\\ ref\ frame}} = \vec{B}^{outside}_{e^-}(\rho \ge a)\Big|_{\substack{intermed\\ ref\ frame}} + \vec{B}^{outside}_{Cu\ atoms}(\rho \ge a)\Big|_{\substack{intermed\\ ref\ frame}} = +\left(\frac{\mu_o I}{2\pi\rho}\right)\hat{\varphi}}$$

which is precisely the same magnetic field as seen by an observer in either the lab frame or the rest frame of the "gas" of "free"/conduction electrons!

Some care must be taken in evaluating the magnetic Lorentz force $\boxed{\vec{F}^{mag}(\vec{r}) = q\vec{v}\ (\vec{r}) \times \vec{B}^{ext}(\vec{r})}$ acting on each of the two species of charge carriers in this intermediate reference frame:

a.) The "gas" of "free"/conduction electrons are each moving with velocity $\boxed{\vec{v}_{e^-} = -\tfrac{1}{2}\langle v_D\rangle \hat{z}}$ and interact with the magnetic field generated by the copper atoms of the fixed/rigid 3-D lattice of the metal wire (moving in the opposite direction) in this reference frame, $\vec{B}^{inside}_{Cu\ atoms}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}}$:

$$\boxed{\begin{aligned}\vec{F}^{mag}_{e^-}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} &= -e\vec{v}_{e^-} \times \vec{B}^{inside}_{Cu\ atoms}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}}\\ &= -e\left(-\tfrac{1}{2}\langle v_D\rangle \hat{z}\right) \times \tfrac{1}{2}\left(\frac{\mu_o I\rho}{2\pi a^2}\right)\hat{\varphi} = +e\langle v_D\rangle\left(\frac{\mu_o I\rho}{8\pi a^2}\right)(\hat{z} \times \hat{\varphi})\end{aligned}}$$

b.) The copper atoms of the fixed/rigid 3-D lattice of the metal wire are each moving with velocity $\boxed{\vec{v}_{Cu\ atoms} = +\tfrac{1}{2}\langle v_D\rangle \hat{z} = -\vec{v}_{e^-}}$ and interact with the magnetic field generated by the "gas" of "free/conduction electrons (moving in the opposite direction) in this frame, $\vec{B}^{inside}_{e^-}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}}$:

$$\boxed{\begin{aligned}\vec{F}^{mag}_{Cu\ atoms}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} &= +e\vec{v}_{Cu\ atoms} \times \vec{B}^{inside}_{e^-}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}}\\ &= +e\left(+\tfrac{1}{2}\langle v_D\rangle \hat{z}\right) \times \tfrac{1}{2}\left(\frac{\mu_o I\rho}{2\pi a^2}\right)\hat{\varphi} = +e\langle v_D\rangle\left(\frac{\mu_o I\rho}{8\pi a^2}\right)(\hat{z} \times \hat{\varphi})\end{aligned}}$$

Thus we see that the Lorentz force acting on the electrons in the "gas" of "free"/conduction electrons vs. the Lorentz force acting on each of the copper atoms are the same in magnitude and direction:

$$\boxed{\vec{F}^{mag}_{e^-}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} = \vec{F}^{mag}_{Cu\ atoms}(\rho \le a)\Big|_{\substack{intermed\\ ref\ frame}} = +e\langle v_D\rangle\left(\frac{\mu_o I\rho}{8\pi a^2}\right)(\hat{z} \times \hat{\varphi})}$$

in the lab reference frame of the 20 AWG conducting copper wire (which is at rest in the lab frame), with the "gas" of drift/conduction electrons moving with (assumed) constant/uniform drift velocity $\vec{v}_{e^-} = \langle \vec{v}_D \rangle = -v_D \hat{z} \simeq -75 \text{ mm/sec}\,\hat{z}$. As we have just shown above, a macroscopic magnetic field is present in the lab frame, which arises from the collective motion of the "gas" of "free" / conduction electrons flowing down the wire.

It is also very important to note here that the macroscopic electron current $\vec{I}_{e^-}\big|_{lab\ frame}$ is derived from the collective effects of the ensemble of "free" conduction electrons at the microscopic level, that are in motion in the lab frame; it is also very important to note that the macroscopic $\vec{I}_{e^-}\big|_{lab\ frame}$ is explicitly obtained via use of the Superposition Principle in going from the microscopic realm to the macroscopic realm in doing so!

So let us now focus our attention on a single one of these drift/conduction electrons, which is moving with (assumed) constant/uniform drift velocity $\vec{v}_{e^-} = \langle \vec{v}_D \rangle = -v_D \hat{z} \simeq -75 \text{ mm/sec}\,\hat{z}$ and ask what an observer at rest in the lab frame sees happen to this single drift/conduction electron. From the above academic/ivory tower discussion, the observer at rest in the lab frame sees a Lorentz force $\vec{F}^{e^-}_{mag}(\vec{r}) = -e\vec{v}_{e^-}(\vec{r}) \times \vec{B}^{ext}_{lab}(\vec{r})$ acting on this particular isolated/single drift / conduction electron, arising from the "external/applied" magnetic field associated with all of the other drift/conduction electrons! {n.b. This is simply using the Superposition Principle in reverse – *i.e.* partitioning the macroscopic electron current $\vec{I}_{e^-}\big|_{lab\ frame}$ into one electron charge + all the other electrons!}. In the rest frame of this isolated/single drift/conduction electron, an observer sees a macroscopic electric field of $\vec{E}_{rest}(\vec{r}\,') = \vec{v}_{lab}(\vec{r}\,') \times \vec{B}^{ext}_{lab}(\vec{r}\,')$ and a corresponding force acting on this electron of $\vec{F}^{elect}_{rest}(\vec{r}\,') = q\vec{E}_{rest}(\vec{r}\,') = q\vec{v}_{lab}(\vec{r}\,') \times \vec{B}^{ext}_{lab}(\vec{r}\,')$.

Note that for "everyday"/garden-variety steady currents of $I = 1.0$ Amperes flowing in a real metal wire, *e.g.* a 20 AWG copper wire, the number density of "free"/conduction electrons in copper is $n^{Cu}_{e^-} = 8.482 \times 10^{28}/\text{m}^3$ (See e.g. P435 Lecture Notes 21, p.11) and since $I = dQ/dt$ then *e.g.* 1 Ampere of steady/DC current corresponds to 1 Coulomb of electric charge per second passing through {any} perpendicular, cross-sectional area $A_\perp = \pi a^2$ of the copper wire (of radius *a*), and one Coulomb per second corresponds to $Q_{tot}/e = 1.0/1.602 \times 10^{-19} = 6.242 \times 10^{18}$ "free" / conduction electrons per second crossing this area – i.e. a ***huge*** number per second, and thus (conceptually) removing a single one of these electrons from the total current has a negligible effect on any reduction in the macroscopic magnetic field arising from the rest of the $\{6.242 \times 10^{18} - 1\}$ remaining "free"/conduction electrons, each of which contributes to the overall macroscopic magnetic field observed in the lab frame.

Now, there is nothing special/unique associated with any one particular electron associated with the "free"/conduction electrons flowing in the conducting 20 AWG wire, thus, it can be seen that the above discussion applies equally to all/each one of the "free"/conduction electrons.

Thus, in the lab frame, each such conduction electron will feel a Lorentz force acting on it:

$$\boxed{\vec{F}^{e^-}_L(\rho)\Big|_{lab\ frame} = q\vec{v}_D \times \vec{B}^{inside}(\rho \le a)\Big|_{lab\ frame} = -e*\left(-\langle v_D \rangle \hat{z}\right) \times \frac{\mu_o}{2\pi a^2} I\rho\hat{\varphi} = -e\frac{\langle v_D \rangle \mu_o}{2\pi a^2} I\rho\left(-\hat{z} \times \hat{\varphi}\right)}$$

$$\boxed{\vec{F}_L^{e^-}(\rho)\Big|_{lab\ frame} = -e\frac{\langle v_D\rangle \mu_o}{2\pi a^2} I\rho(-\hat{z}\times\hat{\varphi})}$$

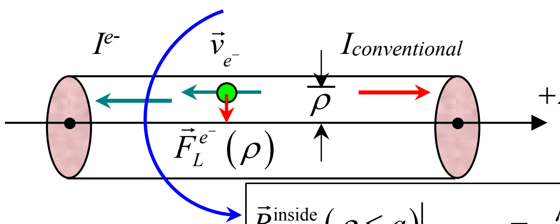
<u>Now</u>: $\boxed{\hat{\varphi} = -\hat{x}\sin\varphi + \hat{y}\cos\varphi}$ <u>and</u>: $\boxed{\hat{x}\times\hat{y}=\hat{z}}$, $\boxed{\hat{y}\times\hat{z}=\hat{x}}$, $\boxed{\hat{z}\times\hat{x}=\hat{y}}$ *etc.*

<u>Thus</u>: $\boxed{-\hat{z}\times\hat{\varphi} = +\hat{z}\times\hat{x}\sin\varphi - \hat{z}\times\hat{y}\cos\varphi = +\hat{y}\sin\varphi + \hat{x}\cos\varphi = \hat{x}\cos\varphi + \hat{y}\sin\varphi}$

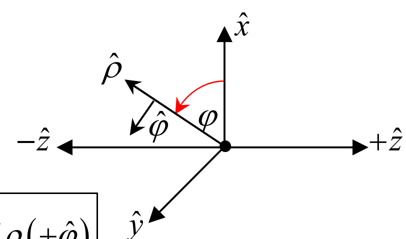
<u>But</u>: $\boxed{\hat{\rho} = \hat{x}\cos\varphi + \hat{y}\sin\varphi}$ $\therefore \boxed{\hat{\rho} = (-\hat{z}\times\hat{\varphi})}$

$\therefore$ $\boxed{\vec{F}_L^{e^-}(\rho)\Big|_{lab\ frame} = -e\frac{\langle v_D\rangle \mu_o I}{2\pi a^2}\rho\hat{\rho} = e\frac{\langle v_D\rangle \mu_o I}{2\pi a^2}\rho(-\hat{\rho})}$

n.b. The Lorentz force acts radially <u>inward</u> on the "gas" of "free"/conduction electrons!



$$\boxed{\vec{B}^{\text{inside}}(\rho\le a)\Big|_{lab\ frame} = \frac{\mu_o}{2\pi a^2} I\rho(+\hat{\varphi})}$$



→ ∃ (i.e. there exists) a radial inward Lorentz force $\vec{F}_L^{e^-}(\rho)$ acting on the "free" / conduction electrons, each moving with (assumed) uniform/constant velocity $-\langle v_D\rangle\hat{z}$ in the lab frame, flowing as a steady/DC macroscopic "conventional" current I (i.e. conventional current flowing in $+\hat{z}$ direction) through the 20 AWG copper wire.

→ ∃ (i.e. there exists) a corresponding, macroscopic radial-inward magnetic <u>pressure</u> $P_{mag}^{e^-}(\rho)$ acting on the "free" / conduction electron "gas", <u>squeezing / compressing</u> it:

$$\boxed{P_{mag}^{e^-}(\rho) = u_{mag}^{inside}(\rho) = \underset{\text{density}}{\overset{\text{magnetic}}{\text{energy}}} = \frac{1}{2\mu_o}\left(B^{inside}(\rho\le a)\right)^2 = \left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{\rho}{a^2}\right)^2}$$

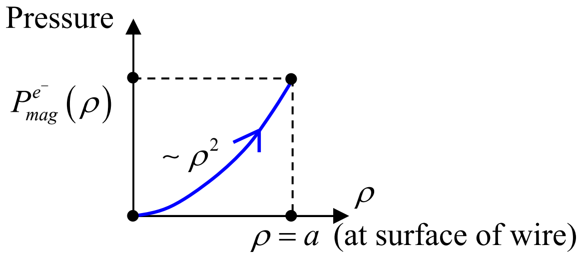
<u>radial inward pressure</u>

(SI Units: Newtons/m^2 (= Pascals) = Joules/m^3)

→ *n.b.* this is the exact same magnetic pressure that we discussed last semester for the cylindrical conducting tube of radius a carrying steady / DC current I:

$$\boxed{P_{mag}(\rho = a) = \left(\frac{\mu_o}{8\pi^2}\right)\frac{I^2}{a^2}}$$

(See P435 Lecture Notes 23, page 18).



The magnetic pressure (self-) acting on the "gas" of "free" / conduction electrons flowing through the 20 AWG copper wire as a steady/DC macroscopic current compresses /squeezes the "free" / conduction electron "gas" radially inward!!!

Using the Superposition Principle (again), we can look at/understand this effect from a slightly different perspective: Imagine partitioning the (assumed uniform) macroscopic electron current density $\vec{J}_{e^-}\Big|_{lab\ frame} = \vec{I}_{e^-}\Big|_{lab\ frame} \Big/ A_\perp^{Cu\ wire}$ into a (very) large number of corresponding filamentary current-carrying wires, all parallel to each other, and each with infinitesimal cross-sectional area $dA_\perp^{Cu\ wire}$. As we learned last semester in P435, from application of the Biot-Savart law in magnetostatics (see P435 Lect. Notes 14, *p*. 12-16), parallel currents attract each other – *i.e.* there is an attractive, "radial-inward" magnetic force acting on pairs of parallel wires carrying steady/DC currents that are flowing in the same direction!

Note that this same phenomena is also operative e.g. in charged and neutral plasmas – and is known as the so-called pinch effect.

Returning to the problem at hand, since the "free" / conduction electron "gas" is compressible, the volume occupied by the electron gas shrinks until opposing internal forces are balanced / in equilibrium (again). Inside the copper wire, there exists (even with no electrical currents present) an internal pressure which is radially outward arising from:

a.) Quantum effects – the confinement / localization of the "free"/conduction electrons to within the spatial confines of the metal conductor (analogous e.g. to the quantum mechanical pressure associated with a particle in a 3-D box); however, here, because electrons are spin-1/2 particles (Fermions) the Pauli Exclusion Principle ("no two identical fermions can simultaneously occupy the same quantum state") is operative here.

b.) Thermal energy associated with each electron ($= \frac{3}{2} k_B T$) associated with the conductor being at finite temperature (here $T = 20^\text{o}$ C) {n.b. Thermionic emission of electrons from metal conductors at finite temperatures is one manifestation of this internal pressure (related to black body radiation / thermal radiation of photons).}

c.) Mutual Coulomb repulsion of the "free" conduction electrons in a compressed electron "gas". Since the "free"/conduction electrons are embedded in a 3-D matrix of positively-charged copper atoms, for which the macroscopic copper wire is overall electrically neutral/has no net electric charge, when no electrical currents are present, there is no net macroscopic Coulomb repulsion for the electrons (and/or the copper atoms) due to mutual cancellation/screening of one species of charge carrier by the other. However, as the "gas" of "free"/conduction electrons is compressed into occupying a smaller volume by the effect of the above radial-inward magnetic pressure, corresponding to an increase in negative-charged electron number density, then there will be an increasing volume electric charge density imbalance $\Delta\rho_\pm \equiv \rho^+_{Cu\ atom} - \rho^-_{e^-}$, and Coulomb repulsion effect to consider.

Thus far, we have not yet investigated the corresponding situation for the copper atoms residing at each of the lattice sites of the 3-D matrix making up the macroscopic copper wire carrying steady/DC conventional current $\vec{I} = \vec{I}_{conv} = +I\hat{z}$. We do this now.

We first consider what is going on in the lab frame, where the wire (and copper atoms) are at rest, but the "free"/conduction electrons are moving with (assumed) uniform/constant velocity $-\langle v_D \rangle \hat{z}$, which collectively generates the macroscopic solenoidal magnetic field:

$\boxed{\vec{B}^{inside}(\rho \le a)\Big|_{lab\ frame} = \dfrac{\mu_o}{2\pi a^2} I\rho\hat{\varphi}}$ and $\boxed{\vec{B}^{outside}(\rho \ge a)\Big|_{lab\ frame} = \dfrac{\mu_o}{2\pi\rho} I\hat{\varphi}}$.

However, all/each of the copper atoms are at rest in the lab frame{neglecting random fluctuations due to finite thermal energy and e.g. possible quantum vibrational effects} and hence no magnetic Lorentz force arises for the copper atoms in this reference frame:

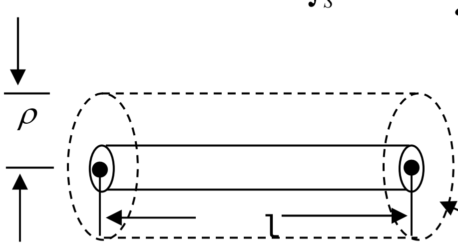
$$\boxed{\vec{F}_L^{Cu\ atom}(\rho)\Big|_{lab\ frame} = +e\underbrace{\vec{v}_L^{Cu\ atom}}_{\equiv 0} \times \vec{B}^{inside}(\rho \le a)\Big|_{lab\ frame} = 0}$$

In the rest frame of the "gas" of "free" conduction electrons, each of the copper atoms are moving with velocity $+\langle v_D \rangle \hat{z}$ and thus generate a

The net result of the magnetic pressure compression of the volume occupied by "free" / conduction "gas" of electrons in e.g. 20 AWG pure copper wire carrying DC / steady current $\vec{I}_{conv} = +1.0\text{ Amp }\hat{z}$ is that the surface of wire becomes positively charged due to absence / deficit of electrons – the inside of the wire, correspondingly, becomes negatively charged due to the excess of electrons there.

- Hence, a radial / outward-pointing electric field $\vec{E}^{outside}(\rho \ge a)$ exists outside the wire, and a radial inward-pointing electric field $\vec{E}^{inside}(\rho \le a)$ also exists inside the wire due to this "surface charge" re-distribution – due to / caused by the radial-inward-pointing dipole layer!! (very thin). Magnetic pressure associated with a steady / DC current I flowing in a long wire.

Using Gauss' Law $\oint_S \vec{E}\cdot d\vec{a} = \dfrac{Q_{encl}}{\varepsilon_o}$: $Q_{encl} = 2\pi a\ell\sigma_+$



$2\pi\rho\ell$ = area of Gaussian surface

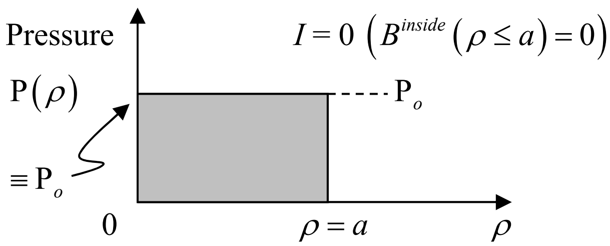
$\vec{E}^{outside}(\rho \ge a) = \dfrac{1}{\varepsilon_o}\left(\dfrac{a}{\rho}\right)\sigma_+\hat{\rho}$

σ = function of magnetic pressure

σ_+ = positive charge / unit area on surface of conductor

If we assume (again) for simplicity's sake that the "free" / conduction electron "gas" in a metal such as (pure) copper behaves as an ideal gas, then it obeys the ideal gas law $PV = Nk_BT$

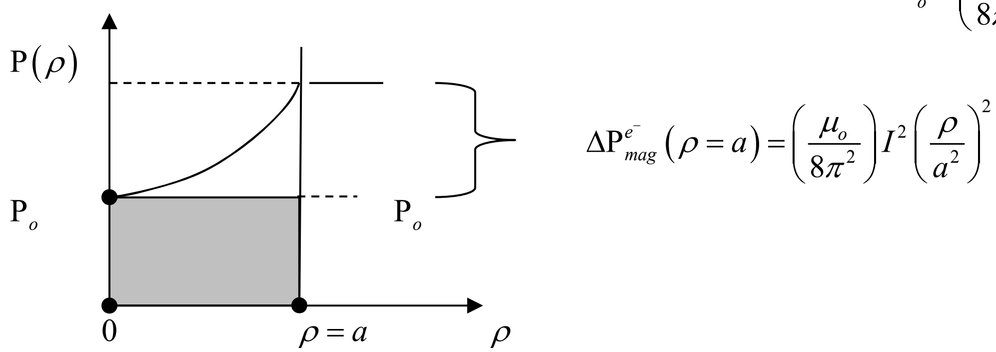
- For no current ($I = 0$) a graph of electron gas pressure vs. radial distance in the wire would be flat / independent of radius:



With a current I flowing in the conductor due to the magnetic pressure

$\Delta \mathrm{P}^{e^-}_{mag}(\rho \le a) = \left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{\rho}{a^2}\right)^2$, the pressure would then be: $\mathrm{P}(\rho \le a) = \mathrm{P}_o + \Delta \mathrm{P}^{e^-}_{mag}(\rho \le a)$

$$= \mathrm{P}_o + \left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{\rho}{a^2}\right)^2$$



Now if: $\mathrm{P}V = Nk_BT \Rightarrow V = \dfrac{Nk_BT}{\mathrm{P}}$

Then: $\dfrac{dV}{d\mathrm{P}} = -Nk_BT \big/ \mathrm{P}^2 \Rightarrow dV = -\dfrac{Nk_BT}{\mathrm{P}^2} d\mathrm{P}$

$\rightarrow$ Change in Volume $\quad \Delta V = \int dV = -Nk_BT \int_{\mathrm{P}_o}^{\mathrm{P}_o + \Delta \mathrm{P}^{e^-}_{mag}(\rho)} \dfrac{d\mathrm{P}}{\mathrm{P}^2}$

Now $\mathrm{P}'V' = \mathrm{P}_o V_o = Nk_BT$ for an ideal gas

$$\therefore \Delta V = -\mathrm{P}_o V_o \int_{\mathrm{P}_o}^{\mathrm{P}_o + \Delta \mathrm{P}^{e^-}_{mag}(\rho)} \frac{d\mathrm{P}}{\mathrm{P}^2}$$

$\mathrm{P}(\rho) = \mathrm{P}_o + \Delta \mathrm{P}^{e^-}_{mag}(\rho) \qquad \Delta \mathrm{P}^{e^-}_{mag}(\rho) = \left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{\rho}{a^2}\right)^2$

$$\mathrm{P}(\rho) = \mathrm{P}_o + \left(\frac{\mu_o}{8\pi^2}\right) I^2 \frac{\rho^2}{a^4}$$

$$\frac{\partial \mathrm{P}(\rho)}{\partial \rho} = 2\left(\frac{\mu_o}{8\pi^2}\right) I^2 \frac{\rho}{a^4} \Rightarrow d\mathrm{P} = 2\left(\frac{\mu_o}{8\pi^2}\right) I^2 \frac{\rho^2}{a^4}\frac{d\rho}{\rho} \qquad d\mathrm{P} = 2\Delta \mathrm{P}_{mag}^{e^-}(\rho)\frac{d\rho}{\rho}$$

@ $\rho = 0$, P ($\rho = 0$)=P$_o$ @ $\rho = a$, $\mathrm{P}(\rho = a) = \mathrm{P}_o + \Delta \mathrm{P}_{mag}^{e^-}(\rho = a)$

$$\therefore \Delta V = -\mathrm{P}_o V_o \int_{\rho=0}^{\rho=a} \frac{2\Delta \mathrm{P}_{mag}^{e^-}(\rho)\, d\rho}{\mathrm{P}^2(\rho)\rho} = -\mathrm{P}_o V_o \int_{\rho=0}^{\rho=a} \frac{2\Delta \mathrm{P}_{mag}^{e^-}(\rho)\, d\rho}{\left[\mathrm{P}_o + \Delta \mathrm{P}_{mag}^{e^-}(\rho)\right]^2 \rho}$$

$$\Delta V = -\mathrm{P}_o V_o \int_{\rho=0}^{\rho=a} \frac{2\left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{\rho^2}{a^4}\right)}{\left[\mathrm{P}_o + \left(\frac{\mu_o}{8\pi}\right)^2 I^2 \left(\frac{\rho^2}{a^4}\right)\right]^2} \frac{d\rho}{\rho}$$

Define: $\alpha \equiv \left(\frac{\mu_o}{8\pi^2}\right) I^2 \frac{1}{a^4}$ and Define: $\beta \equiv \left(\frac{\alpha}{\mathrm{P}_o}\right)$

Then: $$\frac{\Delta V}{V_o} = -2\left(\frac{\alpha}{\mathrm{P}_o}\right)\int_{\rho=0}^{\rho=a} \frac{\rho\, d\rho}{\left[1 + \left(\frac{\alpha}{\mathrm{P}_o}\right)\rho^2\right]^2} = -2\beta \int_{\rho=0}^{\rho=a} \frac{\rho\, d\rho}{\left[1+\beta\rho^2\right]^2}$$

Now define: $u \equiv \left(1+\beta\rho^2\right)$ $\rho = 0$ when $u = 1$

$du = 2\beta\rho\, d\rho$ ρ = a when $u = 1+\beta a^2$

Thus: $$\frac{\Delta V}{V_o} = -\int_{u=1}^{u=1+\beta u^2} \frac{du}{u^2} = +\frac{1}{u}\bigg|_{u=1}^{u=1+\beta a^2} = +\left[\frac{1}{1+\beta a^2} - \frac{1}{1}\right] = -\left[1 - \frac{1}{1+\beta a^2}\right]$$

We assume that (for small / "everyday" laboratory currents I that $\Delta \mathrm{P}_{mag}^{e^-} \ll \mathrm{P}_o$

i.e. $\Delta \mathrm{P}_{mag}^{e^-}$ is a small perturbation on (relatively) large internal pressure P$_o$ $\Rightarrow \beta = \left(\frac{\alpha}{\mathrm{P}_o}\right) \ll 1$

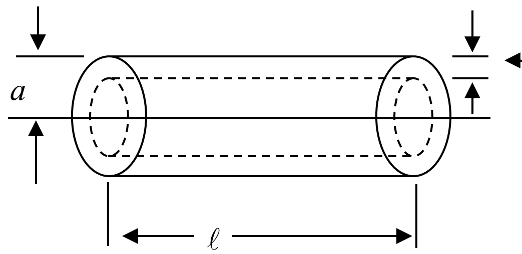
The Taylor-Expand $\left(1 - \frac{1}{1+\beta a^2}\right) \simeq 1 - \left(1-\beta a^2\right) \simeq \beta a^2$ (keeping only up linear term in Taylor Series expansion)

Thus: $$\frac{\Delta V}{V_o} \approx -\beta a^2 = -\left(\frac{\alpha}{\mathrm{P}_o}\right) a^2 = -\frac{\left(\frac{\mu_o}{8\pi^2}\right) I^2 \frac{a^2}{a^4}}{\mathrm{P}_o} = -\frac{\Delta \mathrm{P}_{mag}^{e^-}(\rho = a)}{\mathrm{P}_o}$$

n.b. – sign explicitly indicates decrease in volume due to increase in pressure.

Now $$\frac{\Delta V}{V_o} \underset{\substack{\text{fractional}\\ \text{change in}\\ \text{volume occupied}\\ \text{by electron 'gas'}}}{=} -\frac{\Delta \mathrm{P}_{mag}^{e^-}(\rho = a)}{\mathrm{P}_o} = -\frac{\left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{1}{a^2}\right)}{\mathrm{P}_o}$$

$$= \frac{V' - V_o}{V_o} < 0 \quad \text{(because } V < V_o\text{)}$$



δ = "skin" thickness of the charged layer on surface of wire carrying steady DC current *I*.

$$\frac{\Delta V}{V_o} = -\frac{\left[\pi a^2 - \pi (a-\delta)^2\right]\ell}{\pi a^2 \ell} = -\frac{\left(\pi a^2 - \pi (a-\delta)^2\right)}{\pi a^2} = -\frac{\left(\pi a^2 - \pi\left(a^2 - 2a\delta + \delta^2\right)\right)}{\pi a^2}$$

$$= -\frac{\left(\cancel{\pi a^2} - \cancel{\pi a^2} + 2\pi a\delta - \pi\delta^2\right)}{\pi a^2} = -\frac{\left(2\pi a\delta - \pi\delta^2\right)}{\pi a^2}$$

Assuming $\Delta \mathrm{P}_{mag}^{e^-}(\rho = a) \ll \mathrm{P}_o \Rightarrow \delta \ll a$, then:

$$\therefore \frac{\Delta V}{V_o} = \frac{V' - V_o}{V} \simeq -\left(\frac{2\delta}{a}\right) \simeq -\frac{\left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{1}{a^2}\right)}{\mathrm{P}_o} = -\frac{\Delta \mathrm{P}_{mag}^{e^-}(\rho = a)}{\mathrm{P}_o}$$

δ = thickness of the charged "skin" on surface of current-carrying wire.

Total charge on surface of wire: $Q^+ = +n_{cu} e \Delta V$

Where n_{cu} = number density of copper atoms (# / m^3)

$$Q^+ = +n_{cu} e \Delta V \simeq n_{cu} e \left(\frac{2\delta}{a}\right) V_o = n_u e \left[\frac{\Delta \mathrm{P}_{mag}^{e^-}(\rho = a)}{\mathrm{P}_o}\right]$$

$Q^+ = +n_{cu} e * (2\pi\delta a\ell)$ $\qquad = \pi a^2 \ell$ = volume of copper wire carrying current *I*

$n_{cu} = n_{cu}^{e^-}$ (1 conduction electron / copper atom) ≈ 8.5 x 10^{28} / m^3

With: $\delta \simeq \left(\frac{a}{2}\right)\left(\frac{\Delta \mathrm{P}_{mag}^{(\rho = a)}}{\mathrm{P}_o}\right) = \left(\frac{a}{2}\right) \frac{\left(\frac{\mu_o}{8\pi^2}\right) I^2 \left(\frac{1}{a^2}\right)}{\mathrm{P}_o}$

Assuming the thickness of the positive-charged surface of the conductor to be very thin, i.e. $\delta \ll a$ = radius of wire, since we assumed $\Delta \mathrm{P}_{mag}^{e^-}(\rho) \le \mathrm{P}_o$ = internal pressure of electron gas.

Then surface charge on surface of 20 AWG copper wire carrying steady / DC current *I* will be:

$$\sigma_+ = \frac{Q^+}{A_{cyl}} = \frac{Q^+}{2\pi a\ell} = \frac{n_{cu}e * \cancel{2\pi}\,\delta\,\cancel{a\ell}}{\cancel{2\pi}\,\cancel{a\ell}} = n_{cu}e\delta\,\left(\frac{coulombs}{m^2}\right)$$

Where: $$\delta = \left(\frac{a}{2}\right)\frac{\Delta \text{P}^{e^-}_{mag}(\rho = a)}{\text{P}_o} = \left(\frac{a}{2}\right)\frac{\left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{a^2}\right)}{\text{P}_o}$$

$$\sigma_+ = \frac{Q^+}{A_{cyl}} \simeq n_{cu}e\delta \simeq n_{cu}e\left(\frac{a}{2}\right)\frac{\Delta \text{P}^{e^-}_{mag}(\rho = a)}{\text{P}_o} = n_{cu}e\left(\frac{a}{2}\right)\frac{\left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{a^2}\right)}{\text{P}_o}$$

Then, radial-outward electric field due to this +*ve* charge on the surface of the wire carrying steady / DC current *I* is:

$$\vec{\text{E}}^{outside}(\rho \ge a) = \frac{1}{\varepsilon_o}\left(\frac{a}{\rho}\right)\sigma_+\hat{\rho} \simeq \frac{1}{\varepsilon_o}\left(\frac{a}{\rho}\right)n_{cu}e\delta\hat{\rho} = \frac{1}{\varepsilon_o}\left(\frac{a}{\rho}\right)n_{cu}e\left(\frac{a}{2}\right)\frac{\Delta \text{P}^{e^-}_{mag}(\rho = a)}{\text{P}_o}$$

$$= \frac{1}{\varepsilon_o}\left(\frac{\cancel{a}}{\rho}\right)n_{cu}e\left(\frac{\cancel{a}}{2}\right)\frac{\left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{\cancel{a^2}}\right)}{\text{P}_o}\hat{\rho}$$

$$\vec{\text{E}}^{outside}(\rho \ge a) = \frac{1}{16\pi^2}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\left(\frac{I^2}{\text{P}_o}\right)\frac{1}{\rho}\hat{\rho} \quad \text{Volts / m}$$

Then Poynting's Vector for $\rho \ge a$ is: $$S^{outside}(\rho \ge a) = \frac{1}{\mu_o}\vec{\text{E}}^{outside}(\rho \ge a) \times \vec{B}^{outside}(\rho \ge a)$$

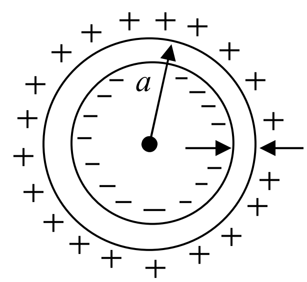
With: $\vec{B}^{outside}(\rho \ge a) = \dfrac{\mu_o}{2\pi\rho}I\hat{\varphi}$ Tesla

$$\vec{S}^{outside}(\rho \ge a) \simeq \frac{1}{16\pi^2}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\left(\frac{I^2}{\text{P}_o}\right)\frac{1}{\rho}\hat{\rho} \times \left(\frac{\mu_o}{2\pi\rho}\right)I\hat{\varphi} = \frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\frac{I^3}{\text{P}_o\rho^2}\hat{\rho}\times\hat{\varphi}$$

Now $\hat{\rho}\times\hat{\varphi} = +\hat{z}$ $\quad\rightarrow\quad$ $\vec{S}^{outside}(\rho \ge a) \simeq \dfrac{1}{32\pi^3}\left(\dfrac{\mu_o}{\varepsilon_o}\right)n_{cu}e\dfrac{I^3}{\text{P}_o\rho^2}\hat{z} \quad \dfrac{Watts}{m^2}$

Poynting's Vector for $\rho \ge a$: $$\vec{S}^{outside}(\rho \ge a) \simeq \frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\frac{I^3}{\text{P}_o\rho^2}\hat{z}$$

Now the electric field just inside the conducting wire, in the region between the dipole layer:



We can model ≈ as the $\vec{E}$-field between inner-outer conductors of a coaxial capacitor with inner radius a- δ and outer radius a with δ<<a.

Again, using Gauss' Law: $\oint_S \vec{E}\cdot\vec{da} = \dfrac{Q_{encl}^{-}}{\varepsilon_o}$

Since δ<<a and $(a-\delta) \approx a$, then: $\left|Q_{encl}^{+}(\rho=a)\right| \simeq \left|Q_{encl}^{-}(\rho=a-\delta)\right|$

i.e. $\left|\sigma^{+}(\rho=a)\right| \simeq \left|\sigma^{-}(\rho=a-\delta)\right|$

Then: $\vec{E}^{inside}(a-\delta\le\rho\le a) = -\dfrac{Q_{encl}^{-}}{2\pi\rho\varepsilon_o}\hat{\rho} = -\dfrac{\cancel{2\pi}(a-\delta)\sigma^{-}}{\cancel{2\pi}\rho\varepsilon_o}\hat{\rho} = -\dfrac{1}{\varepsilon_o}\left(\dfrac{a-\delta}{\rho}\right)\sigma_{-}\hat{\rho}$

Approximating then: $\vec{E}^{inside}(a-\delta\le\rho\le a) = -\dfrac{1}{\varepsilon_o}\left(\dfrac{a}{\rho}\right)\sigma_{-}\hat{\rho}$ and $|\sigma_{+}| \simeq |\sigma_{-}| = n_{cu}e\delta$

[a- $\delta \approx$ a (δ<<a)]

$$= n_{cu}e\left(\frac{a}{2}\right)\frac{\Delta P_{mag}^{e^-}(\rho=a)}{P_o}$$

$$= n_{cu}e\left(\frac{a}{2}\right)\frac{\left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{a^2}\right)}{P_o}$$

$$\therefore \vec{E}^{inside}(a-\delta\le\rho\le a) \simeq -\frac{1}{\varepsilon_o}\left(\frac{\cancel{a}}{\rho}\right)n_{cu}e\left(\frac{\cancel{a}}{2}\right)\frac{\left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{\cancel{a^2}}\right)}{P_o}\hat{\rho}$$

n.b. This is exactly same form as $\vec{E}^{outside}(\rho\ge a)$ except radially inward!!

$\rightarrow$ $\vec{E}^{inside}(a-\delta\le\rho\le a) \simeq -\dfrac{1}{16\pi^2}\left(\dfrac{\mu_o}{\varepsilon_o}\right)n_{cu}e\dfrac{I^2}{P_o\rho}\hat{\rho}$ Volts/m points radially inward in $a-\delta\le\rho\le a$ region

Then Poynting's Vector for $a-\delta \le \rho \le a$ is:

$$\vec{S}^{inside}(a-\delta \le \rho \le a) = \frac{1}{\mu_o}\vec{E}^{inside}(a-\delta \le \rho \le a)\times\vec{B}^{inside}(a-\delta \le \rho \le a)$$

$$\vec{S}^{inside}(a-\delta \le \rho \le a) = -\frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\frac{I^3}{P_o\rho^2}\hat{z} \qquad \text{Watts/m}^2$$

Points opposite direction to $\vec{S}^{outside}(\rho \ge a)$ and points opposite direction to current flow.

Inside the +– dipole layer, *i.e.* $\rho \le (a-\delta)$ we assume (*i.e.* it can be shown) that $\vec{E}^{inside}(\rho \le (a-\delta)) = 0$ as far as radial electric field is concerned.

(We know that longitudinal $\vec{E}$-field: $\vec{E}^{longitudinal}(\rho \le (a-\delta)) = \frac{\Delta V_{batt}}{\ell}\hat{z} = \vec{J}/\sigma_c$ obviously is <u>not</u> zero!)

Gives rise to: $\vec{S}^{inside}(\rho \le (a-\delta)) = \frac{1}{\mu_o}\vec{E}^{longitudinal}(\rho \le (a-\delta))\times\vec{B}^{inside}(\rho \le (a-\delta))$

$\vec{S}^{inside}(\rho \le (a-\delta)) = -\frac{\Delta V_{batt} * I}{2\pi a^2 \ell}\rho\hat{\rho}$ Radially inward flux of EM energy due to Joule heating / Ohmic loss in wire)

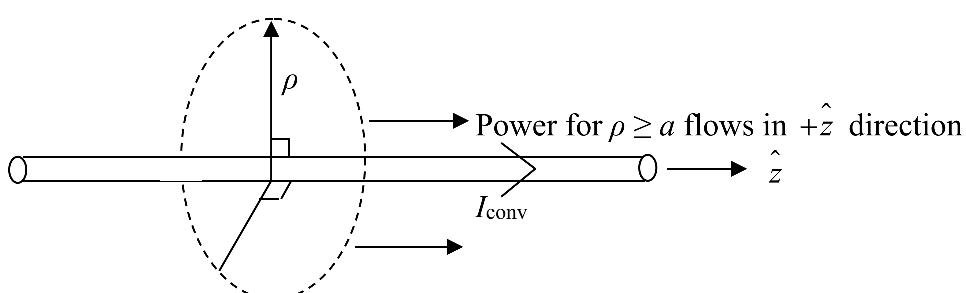
Total Power in each region: $P = \int_S \vec{S}\cdot\vec{da}$

Region outside wire $\rho \ge a$:

$$P^{outside}(\rho \ge a) = \int_{\rho=a}^{\rho=\infty}\vec{S}^{outside}(\rho \ge a)\cdot\underbrace{\vec{da}}_{2\pi\rho d\rho} \simeq \frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\frac{I^3}{P_o}\ell n\left(\frac{\infty}{a}\right) \quad \text{Watts}$$

→ Logarithmically divergent because (infinite area) → Main power transport!!
→ *n.b.* we used formulas for ∞-long wires in this calculation

Just consider Gaussian plane ⊥ to wire:



Dipole layer region $a-\delta \le \rho \le a$:

$$P^{inside}((a-\delta) \le \rho \le a) = \int_{\rho=a-\delta}^{\rho=a}\vec{S}^{inside}((a-\delta) \le \rho \le a)\cdot\vec{da} \simeq \frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\frac{I^3}{P_o}\ell n\left(\frac{a}{a-\delta}\right)$$

$$P^{inside}((a-\delta) \le \rho \le a) \simeq \frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right)n_{cu}e\frac{I^3}{P_o}\underbrace{\ell n\left(\frac{a}{a-\delta}\right)}$$

$$= \ell n\left[a\left(\frac{1}{1-\varepsilon}\right)\right] \text{ with } \left(\varepsilon \equiv \delta/a \ll 1\right)$$

Now Taylor's series expansion of $\ell n\left(\dfrac{a}{a-\delta}\right) \simeq \left(\dfrac{\delta}{a}\right)$ for $\delta \ll a$

$$\therefore \mathrm{P}^{inside}\left(\left(a-\delta\right) \le \rho \le a\right) \simeq \frac{1}{32\pi^3}\left(\frac{\mu_o}{\varepsilon_o}\right) n_{cu} e \frac{I^3}{\mathrm{P}_o}\left(\frac{\delta}{a}\right)$$

Power in $a-\delta \le \rho \le a$ region flows in $-\hat{z}$ direction!!

Note that: $\mathrm{P}^{inside}\left(\left(a-\delta\right) \le \rho \le a\right) <<< \mathrm{P}^{outside}\left(\rho \ge a\right)$

Finally, inside dipole layer region $\rho \le \left(a-\delta\right)$:

$\mathrm{P}\left(\rho \le \left(a-\delta\right)\right) = \Delta V_{batt} * I = I^2 R_{wire}$ = Power dissipated (flows radially inward) due to Joule heating / Ohmic power losses.

$\rightarrow$ Main power transport in current-carrying wire is in / due to EM fields S = E x B outside / external to the wire!!

<u>For completeness' sake:</u>

The calculation of the <u>pressure</u> of the "free" electron "gas" inside the copper wire P_o requires the application of <u>Statistical Mechanics</u> applied to the case of an ideal Fermi Gas (since spin-1/2 electrons are Fermions. Such electrons in e.g. the conduction band of a conducting metal (here, copper) must obey Pauli Exclusion "principle" (no two Fermions of a given quantum system can be in exact same quantum state) and various other quantum mechanical aspects – electrons confined in 3-D lattice of finite spatial extent, etc.

<u>Pressure of ideal electron gas is given by:</u> (see *e.g*. Statistical Mechanics by R.K. Pathria, Pergamon Press, p. 221 (1972))

$$\mathrm{P}_o = \frac{2}{5} n\varepsilon_F\left[1 + \frac{5\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2 + \ldots\right]$$

Ground-state or "zero-point" (T = 0) pressure (purely quantum mechanical in nature!)

$$\varepsilon_F = \text{Fermi Energy} = \left(\frac{6\pi^2 n}{g}\right)^{2/3} \frac{\hbar^2}{2m}$$

n = # density (# / m^3) $\qquad \hbar = \dfrac{h}{2\pi} = \dfrac{\text{Planck's Constant}}{2\pi}$

g = 2 (spin up, spin down) for electrons $\qquad m$ = electron mass (here) but inside metal $\neq m_e$

For pure copper, $\varepsilon_F = 7.0\ eV$ (corresponds to Fermi Temperature)

$n^{cu}_{e^-} = 8.5\times10^{28}\ /\ m^3$

$\varepsilon_F = 1.602\times7\times10^{-19}\ Joules$

$= 1.1214 \times 10^{-18}\ Joules$

$\mathrm{P}^{cu}_o \simeq 3.81276\times10^{10}\ \dfrac{\text{Newtons}}{\text{m}^2}$ (Pascal's)

$1eV = 1.602 \times 10^{-19}$ Joules

$T_F = 8.16\times10^4\ Kelvin$

$\varepsilon_F = k_B T_F$ or $T_F = \varepsilon_F / k_B$

k_B = Boltzman's Constant

$k_B = 1.3806 \times 10^{-23}$ Joules / Kelvin

$k_B = 8.617 \times 10^{-5}\ eV$ / Kelvin

$$\mathrm{P}^{cu}_o = 3.81276\times10^{10}\ \frac{\text{Newtons}}{\text{m}^2}$$

δ = Thickness of +*ve* charged "skin" on surface of current-carrying wire of radius a

$$\delta \approx \left(\frac{a}{2}\right)\left[\frac{\Delta\mathrm{P}_{mag}(\rho=a)}{\mathrm{P}_o}\right] = \left(\frac{a}{2}\right)\left[\frac{\left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{a^2}\right)}{\mathrm{P}_o}\right]$$

$$a = \frac{D(20AWG)}{2} = \frac{1}{2}0.81280\ \text{mm} = 0.4064\ \text{mm}$$

$a = 4.064 \times 10^{-4}$ m for 20 AWG pure copper wire

$$\Delta\mathrm{P}_{mag}(\rho=a) = \left(\frac{\mu_o}{8\pi^2}\right)I^2\left(\frac{1}{a^2}\right)$$

I = 1.0 Ampere

$$\Delta\mathrm{P}_{mag}(\rho=a) = \frac{4\pi\times10^{-7}}{8\pi^2}1^2\left(\frac{1}{4.064\times10^{-4}}\right)^2$$

$\mu_o = 4\pi\times10^{-7}$ Hy / m (= N / A^2)

$\varepsilon_o = 8.85\times10^{-12}$ Farads / m

$$\Delta\mathrm{P}_{mag}(\rho=a) = 0.09636353\ Newtons/m^2 \text{ (= Pascals)}$$

$$\Rightarrow \left[\frac{\Delta\mathrm{P}_{mag}(\rho=a)}{\mathrm{P}_o}\right] = \frac{0.09636353}{3.81276\times10^{10}} = 2.527395\times10^{-12}$$

$$\Rightarrow \delta \simeq \left(\frac{a}{2}\right)\left[\frac{\Delta\mathrm{P}_{mag}(\rho=a)}{\mathrm{P}_o}\right] = 5.13567\times10^{-16}\,m$$

Smaller than the diameter of ? → 0.51367 fm 1 fm = 10^{-15} m

$Q^+(\rho=a) \simeq 2\pi n_{cu} e\delta a\ell$ $a = 4.064 \times 10^{-4}$ m $\ell = 0.5m$

$\simeq 8.9286\times10^{-9}$ Coulombs $n_{cu} = 8.5 \times 10^{28}$ / m^3 $e = 1.602 \times 10^{-19}$ coul.

$$\sigma^+(\rho=a) = Q^+ / A_{\substack{surface\\ wire}} = \frac{Q^+}{2\pi a\ell} \simeq n_{cu} e\delta \simeq 6.9845\times10^{-6}\ \text{Coulombs / m}^2$$

$$A_{\substack{surface\\wire}}\left(\substack{a=4.064\times10^{-4}\,m\\ \ell=0.5m}\right)=2\pi a\ell=1.276743\times10^{-3}\,\text{m}^2$$

At $\rho = a$: $\vec{\mathrm{E}}^{outside}(\rho=a)=\dfrac{\sigma_+}{\varepsilon_o}\hat{\rho}=\dfrac{6.9845\times10^{-6}\,Coul/m^2}{8.85\times10^{-12}\,Farads/m}=7.902\times10^5\,Volts/m\left(\dfrac{a}{\rho}\right)\hat{\rho}$

$\left|Q^-(\rho=a-\delta)\right|=\left|Q^+(\rho=a)\right|$ $\quad\left|\sigma^-(\rho=a-\delta)\right|\simeq\left|\sigma^+(\rho=a)\right|$ (for $\delta << a$)

$\left|\vec{\mathrm{E}}^{inside}(\rho=a)\right|=\left|\vec{\mathrm{E}}^{outside}(\rho=a)\right|$ (for $\delta << a$)

At $\rho = a$: $\vec{S}^{outside}(\rho=a)=\dfrac{1}{32\pi^3}\left(\dfrac{\mu_o}{\varepsilon_o}\right)n_{cu}e\dfrac{I^3}{\mathrm{P}_o a^2}\hat{z}=3.0946\times10^8\ \text{Watts}/\text{m}^2\ \hat{z}$

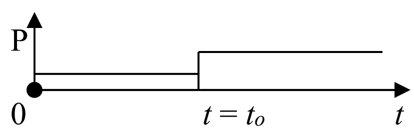
→ Thus static EM fields (in external proximity to wire) carries / transports DC electrical power along a wire. Microscopically, EM power is transported / carried by (very large numbers of) virtual photons in $+\hat{z}$ direction!!

It makes complete / intuitive sense that (even) DC electrical power is carried by virtual photons and not directly by "free" / conduction electrons flowing in the wire.

WHY??

Drift electrons have very low speeds $\langle V_D\rangle\simeq 75\,\mu m/\sec$ (in copper) yet (bare) wires can easily transport abrupt changes in DC electrical power

(*e.g.* a step function



at typical speeds $V_{prop}\approx 50-60\%$ speed of light C. ($C = 3 \times 10^8$ m/s)
This can only occur via power transport by virtual photons associated with (external) EM fields $S^{outside}(\rho\ge a)=\dfrac{1}{\mu_o}\vec{\mathrm{E}}^{outside}(\rho\ge a)\times\vec{B}^{outside}(\rho\ge a)$ - mostly within outside / external proximity to wire.

Note that: $V_{prop}^{wire}\simeq 1\big/\sqrt{\mathcal{L}\mathcal{C}}$

$\mathcal{L}$ = Inductance / unit length of wire, typically $\mathcal{O}$ (≈ 0.4 μHy/foot)
$\mathcal{C}$ = Capacitance / unit length of wire, typically $\mathcal{O}$ (≈ 10 ρf/foot)

$$\mathcal{C}\approx\frac{10\rho f}{ft}\simeq\frac{10\rho f}{30\text{cm}}=\frac{10\rho f}{0.3\text{m}}\simeq 33\ \rho f/m=33\times10^{-12}\ farads/\text{m}$$

$$\mathcal{L}\simeq 0.4\ \mu Hy\big/\text{ft}=\frac{0.4\mu Hy}{30\text{cm}}=\frac{0.4\mu Hy}{0.3\text{m}}\simeq1.33\,\mu H\big/\text{m}=1.33\times10^{-6}\,Henrys\big/\text{m}$$

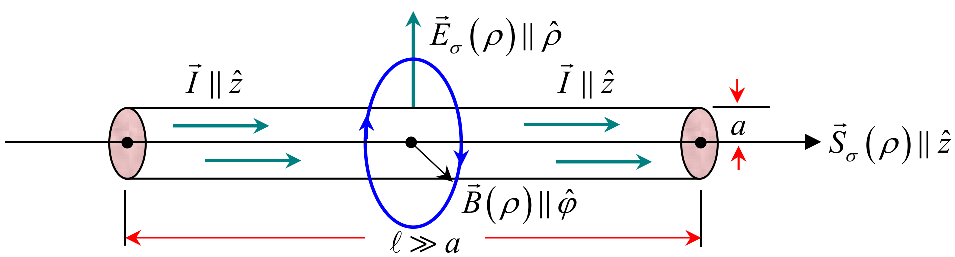
$$\therefore V_{prop} = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{1}{\sqrt{1.33\times10^{-6} * 33\times10^{-12}}} \simeq 1.508\times10^{8} \text{ meters}/\text{sec}$$

$$\beta_{prop} \equiv \frac{V_{prop}}{C} = \frac{1.508\times10^{8} \text{ m}/\text{s}}{3\times10^{8} \text{ m}/\text{s}} \simeq 50.25\% \text{ speed of light}$$

additionally sets up/creates a positive electric charge density on the outer surface of the (long) wire. This positive surface charge density $+\sigma$ on the outer surface of the (long) wire has associated with it a radially outward-pointing electric field, $\vec{E}_\sigma(\rho) = E_\sigma(\rho)\hat{\rho}$. It is *this* radial-outward pointing electric field, which when crossed with the azimuthal magnetic field $\vec{B}(\rho) = B(\rho)\hat{\varphi}$ associated with the steady/DC current I flowing down the (long) conducting wire is responsible for creating this "additional" Poynting's vector:

$$\boxed{\vec{S}_\sigma(\rho) \equiv \tfrac{1}{\mu_o}\vec{E}_\sigma(\rho)\times\vec{B}(\rho) = \tfrac{1}{\mu_o}E_\sigma(\rho)B(\rho)\underbrace{(\hat{\rho}\times\hat{\varphi})}_{=+\hat{z}} = +S_\sigma(\rho)\hat{z}}$$

which in turn is responsible for transporting the vast bulk of the *EM* power from the electrical power source (here, a battery) to the load (here, a resistor) along the (long) conducting wires of the circuit!!!



We can use (the integral form of) Gauss' law to determine the nature of this radial electric field, in terms of the +ve surface charge density, $+\sigma$:

$$\oint_S \vec{E}_\sigma(\vec{r})\cdot d\vec{a} = Q_{encl}/\varepsilon_o$$

Using an imaginary cylindrical Gaussian enclosing surface, we see that for $\rho < a$, $\vec{E}_\sigma(\rho < a) = 0$ because $Q_{encl} = 0 \ \ \rho < a$. For $\rho \geq a$, due to the aspects of cylindrical symmetry associated with this problem, we see that {far from the ends of the long wire}:

$$\vec{E}_\sigma(\rho \geq a) = +\frac{Q_{encl}}{A_{\substack{Gaussian\\cylinder}}\varepsilon_o}\hat{\rho} = +\frac{2\pi a\ell\sigma}{2\pi\rho\ell\varepsilon_o}\hat{\rho} = +\frac{\sigma}{\varepsilon_o}\left(\frac{a}{\rho}\right)\hat{\rho}$$

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19.) J.D. Jackson, "*Surface Charges on Circuit Wires and Resistors Play Three Roles*", American Journal of Physics, Vol. 64, No. 7, p. 855-870, July, 1996.

20.) N.W. Preyer, "*Surface Charges and Fields of Simple Circuits*", American Journal of Physics, Vol. 68, No. 11, p. 1002-1006, Nov., 2000.

Appendix A

American Wire Gauge (AWG) & Metric Gauge Wire Sizes

AWG Wire Sizes (see table below)

AWG: In the American Wire Gauge (AWG), diameters can be calculated by applying the formula: D(AWG) = 0.005 * 92 $^{((36\text{-AWG})/39)}$ inch. For the 00, 000, 0000 etc. gauges you use -1, -2, -3, which makes more sense mathematically than "double nought." This means that in American Wire Gauge every 6 gauge decrease gives a doubling of the wire diameter, and every 3 gauge decrease doubles the wire cross sectional area – just like calculating dB's in signal levels.

Metric Wire Gauges (see table below)

Metric Gauge: In the Metric Gauge scale, the gauge is 10 times the diameter in millimeters, thus a 50 gauge metric wire would be 5 mm in diameter. Note that in AWG the diameter goes up as the gauge goes down. Metric is the opposite. Probably because of this confusion, most of the time metric sized wire is specified in millimeters rather than metric gauges.

Load Carrying Capacities (see table below)

The following chart is a guideline of "ampacity", or copper wire current-carrying capacity following the *Handbook of Electronic Tables and Formulas* for American Wire Gauge. As you might guess, the rated "ampacities" are just a rule of thumb. In careful engineering the insulation temperature limit, thickness, thermal conductivity, and air convection and temperature should all be taken into account. The Maximum Amps for Power Transmission uses the 700 circular mils per amp rule, which is very conservative. The Maximum Amps for Chassis Wiring is also a conservative rating, but is meant for wiring in air, and not in a bundle. For short lengths of wire, such as is used in battery packs, you should trade off the resistance and load with size, weight, and flexibility.

AWG Gauge	**Diameter (Inches)**	**Diameter (mm)**	**Ohms per 1000′ (@ T=20ºC)**	**Ohms per km (@ T=20ºC)**	**Max amps for chassis wiring**	**Max amps for power X-mission**
0000	0.4600	11.6840	0.0490	0.160720	380	302
000	0.4096	10.40384	0.0618	0.202704	328	239
00	0.3648	9.26592	0.0779	0.255512	283	190
0	0.3249	8.25246	0.0983	0.322424	245	150
1	0.2893	7.34822	0.1239	0.406392	211	119
2	0.2576	6.54304	0.1563	0.512664	181	94
3	0.2294	5.82676	0.1970	0.646160	158	75
4	0.2043	5.18922	0.2485	0.815080	135	60
5	0.1819	4.62026	0.3133	1.027624	118	47
6	0.1620	4.11480	0.3951	1.295928	101	37
7	0.1443	3.66522	0.4982	1.634096	89	30
8	0.1285	3.26390	0.6282	2.060496	73	24
9	0.1144	2.90576	0.7921	2.598088	64	19
10	0.1019	2.58826	0.9989	3.276392	55	15
11	0.0907	2.30378	1.2600	4.132800	47	12
12	0.0808	2.05232	1.5880	5.208640	41	9.3
13	0.0720	1.82880	2.0030	6.569840	35	7.4
14	0.0641	1.62814	2.5250	8.282000	32	5.9
15	0.0571	1.45034	3.1840	10.44352	28	4.7
16	0.0508	1.29032	4.0160	13.17248	22	3.7
17	0.0453	1.15062	5.0640	16.60992	19	2.9
18	0.0403	1.02362	6.3850	20.94280	16	2.3
19	0.0359	0.91186	8.0510	26.40728	14	1.8
20	0.0320	0.81280	10.150	33.29200	11	1.5
21	0.0285	0.72390	12.800	41.98400	9	1.2
22	0.0254	0.64516	16.140	52.93920	7	0.92
23	0.0226	0.57404	20.36	66.78080	4.7	0.729
24	0.0201	0.51054	25.67	84.19760	3.5	0.577
25	0.0179	0.45466	32.37	106.1736	2.7	0.457
26	0.0159	0.40386	40.81	133.8568	2.2	0.361
27	0.0142	0.36068	51.47	168.8216	1.7	0.288
28	0.0126	0.32004	64.9	212.8720	1.4	0.226
29	0.0113	0.28702	81.83	268.4024	1.2	0.182
30	0.0100	0.254	103.2	338.4960	0.86	0.142
31	0.0089	0.22606	130.1	426.7280	0.700	0.1130
32	0.0080	0.2032	164.1	538.2480	0.530	0.0910
Metric 2.0	0.00787	0.200	169.4	555.6100	0.510	0.0880
33	0.00710	0.18034	206.9	678.6320	0.430	0.0720
Metric 1.8	0.00709	0.18000	207.5	680.5500	0.430	0.0720
34	0.00630	0.16002	260.9	855.7520	0.330	0.0560
Metric 1.6	0.00630	0.16002	260.9	855.7520	0.330	0.0560
35	0.00560	0.14224	329.0	1079.120	0.270	0.0440
Metric 1.4	0.00551	0.14000	339.0	1114	0.260	0.0430
36	0.00500	0.12700	414.8	1360	0.210	0.0350
Metric 1.25	0.00492	0.12500	428.2	1404	0.200	0.0340
37	0.00450	0.11430	523.1	1715	0.170	0.0289
Metric 1.12	0.00441	0.11200	533.8	1750	0.163	0.0277
38	0.00400	0.10160	659.6	2163	0.130	0.0228
Metric 1	0.00394	0.10000	670.2	2198	0.126	0.0225
39	0.00350	0.08890	831.8	2728	0.110	0.0175
40	0.00310	0.07874	1049	3442	0.090	0.0137
41	0.00280	0.07112	1323	4341		
42	0.00250	0.06350	1659	5443		
43	0.00220	0.05588	2143	7031		
44	0.00200	0.05080	2593	8507		
45	0.00176	0.04470	3348	10984		
46	0.00157	0.03988	4207	13802		
47	0.00140	0.03556	5291	17359		

