Electromagnetic Multipole Moments of Elementary Particles

- In general, a fundamental particle with intrinsic spin ***S*** = 0, ½, 1, …is allowed to have (up to) ***2S+1 CP***-conserving (i.e. ***T***-conserving) ***EM*** moments.

- A spin-***S*** particle *may* have (up to) ***2S+1 CP***-violating (i.e. ***T***-violating) ***EM*** moments.

- Spin ½ Fermions:
  Electric Charge, $Q = e$ (Electric Monopole Moment)
  Magnetic Dipole Moment, $\mu = \frac{1}{2} g e\hbar/m$, $g = 2.0$ (g = g-factor = gyro-magnetic ratio, μ/S)
  {Electric Dipole Moment, $d = \frac{1}{2} e ƛ$, $ƛ = \hbar/mc = \hbar c/mc^2$ ($d = 0$ in SM)}

- Spin 1 Bosons:
  Electric Charge, $Q = e$ (Electric Monopole Moment)
  Magnetic Dipole Moment, $\mu = \frac{1}{2} g e\hbar/m$, $g = 2.0$ (g = g-factor = gyro-magnetic ratio, μ/S)
  {Electric Dipole Moment, $d = \frac{1}{2} e ƛ$, $ƛ = \hbar/mc = \hbar c/mc^2$ ($d = 0$ in SM)}
  Electric Quadrupole Moment, $Q^e = -e ƛ^2$, $ƛ = \hbar/mc = \hbar c/mc^2$
  {Magnetic Quadrupole Moment, $Q^m = -ec ƛ^2$, $ƛ = \hbar/mc = \hbar c/mc^2$ ($Q^m = 0$ in SM)}

- Photon & Z^0 (spin-1 bosons) are each their own anti-particles ⇒ can have no *static* ***EM*** moments!!! Can have *transition* ***EM*** moments (= 0 in SM)!!!

- Electron (spin ½ fermion): $\mu_e = \frac{1}{2} g_e e\hbar/m_e = g_e \mu_{\mathbf{B}} = (2+\Delta\kappa_e)\mu_B$
  $g_e = (2+\Delta\kappa_e) = 2.0023193$, $\Delta\kappa_e = (g_e - 2) = 0.0023193$
  $\mu_{\mathbf{B}} = \frac{1}{2} e\hbar/m_e = 5.78838\times10^{-11}$ MeV/Tesla

- Muon (spin ½ fermion): $\mu_\mu = \frac{1}{2} g_\mu e\hbar/m_\mu = g_\mu \mu_{\mathbf{B}\mu} = (2+\Delta\kappa_\mu)\mu_{B\mu}$
  $g_\mu = (2+\Delta\kappa_\mu) = 2.0023332$, $\Delta\kappa_\mu = (g_\mu - 2) = 0.0023332$

- Proton (spin ½ fermion): $\mu_p = \frac{1}{2} g_p e\hbar/m_p = g_p \mu_{\mathbf{N}} = (2+\Delta\kappa_p)\mu_N$
  $g_p = (2+\Delta\kappa_p) = 5.5856940$, $\Delta\kappa_p = (g_p - 2) = 3.5856940$ ⇒ Not fundamental - composite!!!
  $\mu_{\mathbf{N}} = \frac{1}{2} e\hbar/m_p = 3.15245\times10^{-14}$ MeV/Tesla

- Neutron (spin ½ fermion): $\mu_n = \frac{1}{2} g_n e\hbar/m_p = g_n \mu_{\mathbf{N}} = (2+\Delta\kappa_n)\mu_N$
  $g_n = (2+\Delta\kappa_n) = -3.8260856$, $\Delta\kappa_n = (g_n - 2) = -5.8260856$ ⇒ Not fundamental - composite!!!

- *W*-Boson (spin 1 boson): $\mu_w = \frac{1}{2} g_w e\hbar/m_w = g_w \mu_{\mathbf{w}} = (2+\Delta\kappa_w+\lambda_w)\mu_{\mathbf{w}}$
  $Q^e{}_W = q^e{}_W(-e ƛ^2{}_W) = (1+\Delta\kappa_w-\lambda_w)(-e ƛ^2{}_W)$
  $ƛ_W = \hbar/m_W c = \hbar c/m_W c^2 = 2.45\times10^{-3}$ fm (1 fm = 10^{-15} m)
  {$d_W = \delta_W(e ƛ_W) = (\tilde{\kappa}_w+\tilde{\lambda}_w)(e ƛ_W)$ ($d_W = 0$ in SM)}
  {$Q^m{}_W = q^m{}_W(-ec ƛ^2{}_W) = (\tilde{\kappa}_w-\tilde{\lambda}_w)(-ec ƛ^2{}_W)$ ($Q^m{}_W = 0$ in SM)}