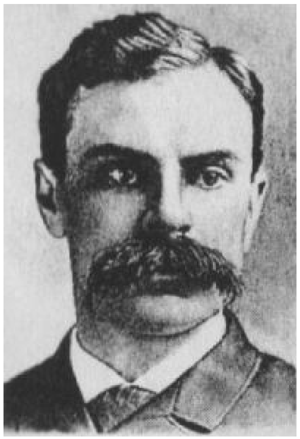
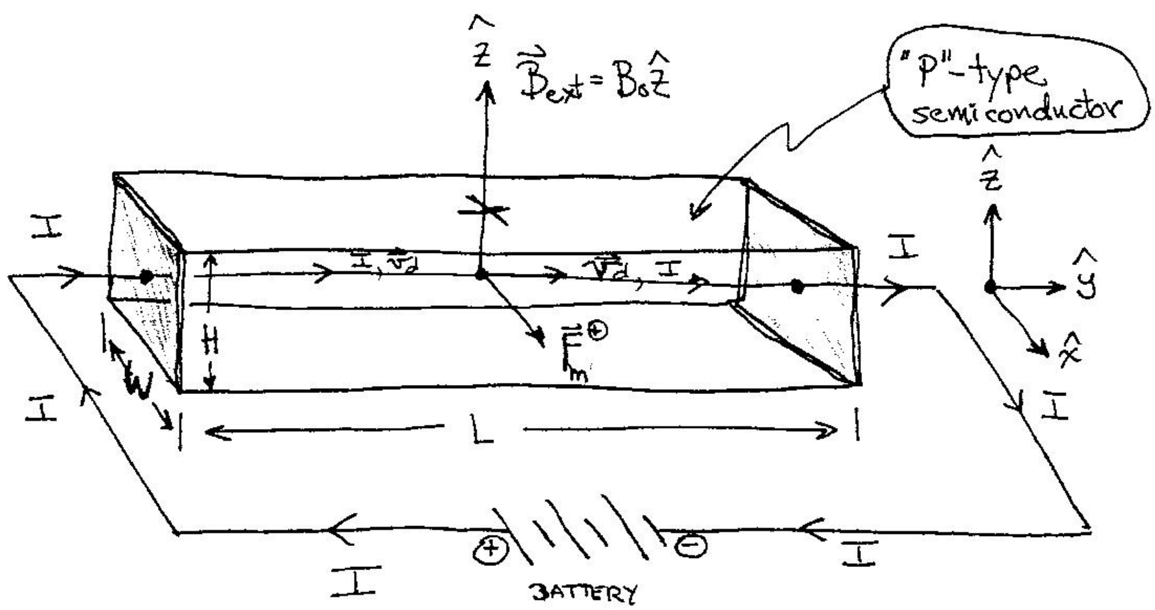
Supplemental Handout #5

The Hall Effect In Semiconducting Materials



In 1879, Edwin Hall, while working on his Ph.D. thesis (Johns Hopkins, Baltimore, MD {thesis advisor: Prof. Henry Rowland (of Rowland Ring fame…)}) discovered the following effect:

Place a (small) rectangular block of (doped) semiconducting material (e.g. silicon or germanium) of dimensions $L \times W \times H$ in a uniform & constant external magnetic field $\vec{B}_{ext} = B_o \hat{z}$ which is $\perp$ to the long axis of the semiconductor. Connect a battery up to the semiconductor across the ends of its long axis such that a steady electrical current I_{free} flows through the semiconductor, parallel to its long axis (i.e. $\vec{I}_{free} \parallel \hat{y}$), as shown in the figure below:



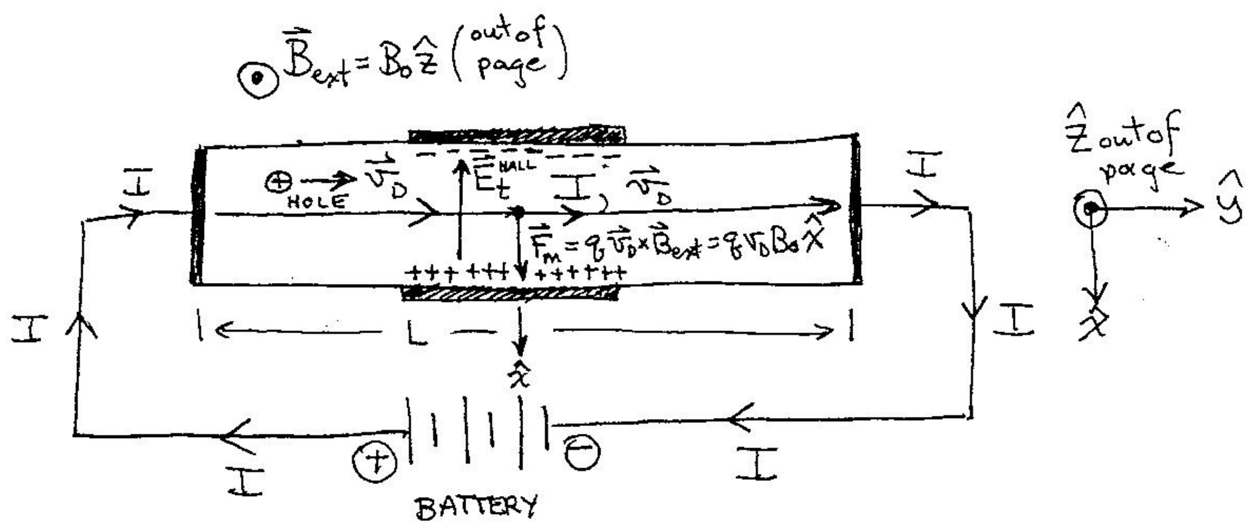
Then because a free current $\vec{I}_{free} = I_{free}\hat{y}$ is flowing in the presence of a $\perp$ applied external magnetic field $\vec{B}_{ext} = B_o \hat{z}$, a Lorentz force $\vec{F}_m = q\vec{v}_D \times \vec{B}_{ext}$ acts on each charge carrier flowing in the semiconductor, which causes a deflection of the charge carriers transverse to both the long axis of the semiconductor ($\hat{y}$ -direction) and to $\vec{B}_{ext} = B_o \hat{z}$, since $\boxed{\vec{F}_m = q\vec{v}_D \times \vec{B}_{ext} = qv_D B_o \hat{x}}$.

Consequently, electric charge builds up on the (here, vertical) sides ($\hat{n}_{side} = \mp\hat{x}$ direction) of the semiconductor, which generates a transverse electric field $\vec{E}_t = -E_t \hat{x}$ across the semiconductor, which in turn prevents/stops further transverse drift of the charge carriers – i.e.

this transverse electric field compensates for the Lorentz force acting on the charge carriers flowing through the semiconducting material. Once this transverse electric field has been established, the current then flows "normally" through the semiconducting material in the presence of both $\vec{B}_{ext}$ and $\vec{E}_t$.

If the (doped) semiconductor is "*p*"-type (as opposed to "*n*"-type) {achieved by doping the semiconductor material with e.g. arsenic or phosphorous, or e.g. boron}, then in a "*p*"-type semiconductor, microscopically, the conduction is due to positive-charged "holes" (= absence of –*ve* charge) in the 3-D crystal lattice of the semiconductor. In an "*n*"-type semiconductor, conduction is due to electrons.

In the situation here, suppose the semiconductor is "*p*"-type; ⇒ conduction is due to +*ve* "holes". Then in a top view of the "*p*"-type semiconductor shows the configuration of fields and forces present:



Thus we see that the transverse electric field in the semiconductor, now known as the Hall field, $\vec{E}_t^{Hall} = -E_t^{Hall}\hat{x}$ that gets set up inside the semiconductor is such that the corresponding electric force on the charge carriers flowing in the semiconductor perfectly balances the Lorentz force, i.e.:

$$\boxed{\vec{F}_{Tot} = \vec{F}_m + \vec{F}_E = +q_{hole}v_D B_o\hat{x} - q_{hole}E_t^{Hall}\hat{x} = 0} \quad \Rightarrow \quad \boxed{\vec{E}_t^{Hall} = -v_D B_o \hat{x}}$$

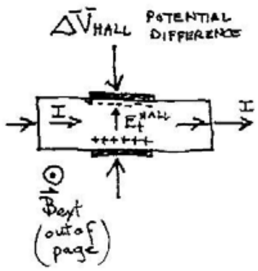
The electric force acting on the charge carrier flowing through the semiconductor due to the transverse electric (Hall) field $\vec{E}_t^{Hall}$ is known as the Hall force, i.e. $\boxed{\vec{F}_E^{Hall} = q_{hole}\vec{E}_t^{Hall}}$. Then $\vec{F}_{Tot} = \vec{F}_m + \vec{F}_E^{Hall} = 0$ or $\boxed{\vec{F}_E^{Hall} = -\vec{F}_m = -q_{hole}v_D B_o\hat{x}}$ (for a "*p*"-type semiconductor).

If the width W of the semiconducting material is much less than its length L (i.e. $W \ll L$) then the transverse electric field (Hall field) $\vec{E}_t^{Hall}$ is $\approx$ uniform/constant across the width of the semiconductor, and thus:

$$\boxed{E_t^{Hall} = \left|\vec{E}_t^{Hall}\right| = \Delta V_{Hall}/W}$$

where ΔV_{Hall} is the potential difference across the transverse dimension (i.e. width) of the semiconductor (here with unit normals $\hat{n}_{side} = \mp\hat{x}$ -directions).

Thus, we see that $\boxed{\Delta V_{Hall} = WE_t^{Hall} = W\left(v_D B_o\right)}$ $\Leftarrow$ known as the Hall voltage.



We can turn this relation around to read: $\boxed{B_o = \Delta V_{Hall}/\left(v_D W\right)}$. What this relation says is, that if we measure the Hall voltage ΔV_{Hall} and the width, W of the semiconducting sample, then if the (mean) drift speed v_D of the carriers flowing through the (doped) semiconducting sample is also known (e.g. from independent experimental measurement(s)), then we can use the Hall effect in (doped) semiconducting material(s) as a means to measure and/or monitor magnetic fields!!!

Today, the available technology is such that it is easy to accurately measure/monitor the Hall voltage ΔV_{Hall} in real time. Measurement of the width, W of the semiconducting sample is a one-time deal. But how does one determine the (mean) drift speed v_D of the carriers flowing through the (doped) semiconducting sample?

If the doping of the semiconductor sample is uniform throughout the sample, then we know that the volume free current density $\vec{J}_{free} = J_{free}\hat{y}$ is uniform throughout the sample.

Then: $\boxed{I_{free} = \vec{J}_{free} \bullet \vec{A}_\perp = J_{free}\left(WH\right)}$ since $\boxed{\vec{A}_\perp = WH\hat{y}}$, i.e. $\vec{J}_{free} \parallel \vec{A}_\perp$.

However, the volume free current density is $\boxed{\vec{J}_{free} = nq\vec{v}_D}$ where n = number density (#/m^3) of charge carriers (holes or electrons in "*p*" or "*n*"-type doped semiconductor material, respectively).

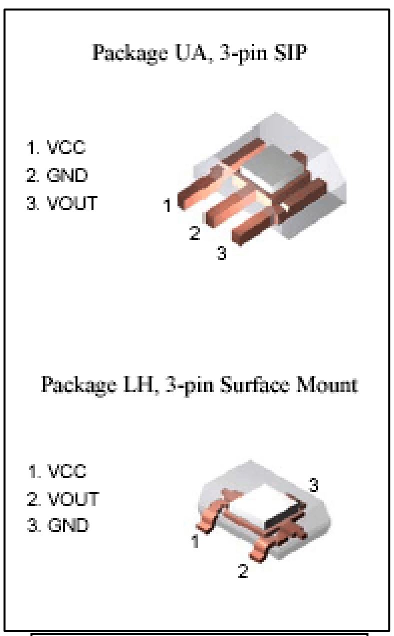
Thus: $\boxed{I_{free} = \vec{J}_{free} \bullet \vec{A}_\perp = J_{free}\left(WH\right) = nqv_D WH}$ $\Rightarrow$ $\boxed{\left(v_D W\right) = \dfrac{I_{free}}{nqH}}$

Hence we see that: $\boxed{B_o = \dfrac{\Delta V_{Hall}}{\left(v_D W\right)} = \dfrac{nqH\Delta V_{Hall}}{I_{free}}}$ or: $\boxed{\Delta V_{Hall} = \dfrac{I_{free}B_o}{nqH} = \dfrac{1}{nq}\left(\dfrac{I_{free}B_o}{H}\right) = R_{Hall}\left(\dfrac{I_{free}B_o}{H}\right)}$

The Hall coefficient associated with the doped semiconductor is $\boxed{R_{Hall} \equiv 1/nq = |\vec{v}_D|/|\vec{J}_{free}|}$ (SI units m^3/Coulomb) which contains all of the detailed condensed matter physics (at the microscopic scale). The number density of charge carriers, n in a semiconducting material depends (significantly) on temperature (at least over a wide temperature range), i.e. $n = n(T)$ and thus the Hall coefficient is also formally temperature dependent $R_{Hall}(T) \equiv 1/n(T)q$.

In practice, a commercial Hall probe that is ultimately to be used e.g. in a laboratory setting for measurement/monitoring of magnetic fields is first absolutely calibrated by measuring ΔV_{Hall} vs. B_o for known values of B_o at a known/measured constant current I_{free} and at the temperature the Hall probe will be routinely used at (nominally room temperature, i.e. T = 20 C). A least-squares straight-line fit to the ΔV_{Hall} vs. B_o data {a linear $y = mx$ relation, where the slope $\boxed{m = R_{Hall} I_{free}/H}$} then enables accurate determination of the Hall coefficient, R_{Hall} {assuming the thickness/height H of the doped semiconducting sample is accurately known/has been accurately measured}.

Once the slope m/Hall coefficient R_{Hall} has been determined, the Hall probe can then be used to measure/monitor magnetic fields simply by measuring/monitoring the Hall voltage $\boxed{\Delta V_{Hall} = mB_o = (R_{Hall} I_{free} B_o)/H}$ and operating of the Hall probe at the same constant current I_{free} that the device was originally calibrated at (commercially-available Hall probes are always accompanied with associated support electronics that automatically provides this, since the Hall probes are originally calibrated in conjunction with the accompanying support electronics).



Allegro-Micro 1321
Linear Hall Sensor

Today the use of Hall-effect devices is widespread – miniature Hall sensors are used seemingly everywhere – e.g. in the automotive industry for speedometers, odometers, tachometers, anti-lock brake systems, transmission control, etc. Digital Hall-effect sensors can be used for proximity switches, brushless DC motors and in automated manufacturing control/monitoring process applications. Hall sensors are also have applications in the computer printer and paper copying industry.

Many companies are making linear and digital Hall sensors today. Linear Hall sensors typically have an output response of ~ 1-5 millvolts/Gauss (recall 1 Gauss = 10^{-4} Tesla), thus they are quite sensitive – the earth's magnetic field (on the surface of the earth) is of order ~ 0.5 Gauss.

