Supplemental Handout #4

$$c^2 = \frac{1}{\varepsilon_o \mu_o}$$

Summary of Electro*statics* & Magneto*statics*

Electro*statics*:

$$\vec{F}_E = q\vec{E} + q\vec{v} \times \vec{B}_{ext}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$

Gauss' Law:

$$\vec{\nabla} \bullet \vec{E} = \frac{1}{\varepsilon_o}\rho_{TOT} = \frac{1}{\varepsilon_o}\left(\rho_{free} + \rho_{bound}\right)$$

$$\Phi_E = \oint_S \vec{E} \bullet d\vec{a} = \frac{1}{\varepsilon_o} Q_{TOT}^{enclosed} = \frac{1}{\varepsilon_o}\left(Q_{free}^{encl.} + Q_{bound}^{encl.}\right)$$

$\vec{\nabla} \times \vec{E} = 0$ always, $\Rightarrow$ $\vec{E} = -\vec{\nabla} V$

Linear Media:

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_o \vec{E} + \vec{\mathrm{P}}$$

$$\varepsilon = \varepsilon_o\left(1 + \chi_e\right), \quad K_e = \varepsilon/\varepsilon_o$$

$$\vec{\mathrm{P}} = \varepsilon_o \chi_e \vec{E}$$

$$\vec{\nabla} \times \vec{D} = \vec{\nabla} \times \vec{\mathrm{P}}$$

$$\rho_{bound} = -\vec{\nabla} \bullet \vec{\mathrm{P}}$$

$$\sigma_{bound} = \vec{\mathrm{P}} \bullet \hat{n}\Big|_{\text{interface}}$$

$$\vec{\nabla} \bullet \vec{D} = \rho_{free} \qquad \vec{\nabla} \bullet \vec{\mathrm{P}} = -\rho_{bound}$$

$$\Phi_D = \oint_S \vec{D} \bullet d\vec{a} = Q_{free}^{encl}$$

$$\Phi_{\mathrm{P}} = \oint_S \vec{\mathrm{P}} \bullet d\vec{a} = -Q_{bound}^{encl}$$

$$\rho_{bound} = -\frac{\chi_e}{\left(1 + \chi_e\right)}\rho_{free}$$

$$\vec{\nabla} \bullet \vec{J}_{free} = -\frac{\partial \rho_{free}}{\partial t}, \quad \vec{\nabla} \bullet \vec{J}_{bound} = -\frac{\partial \rho_{bound}}{\partial t},$$

$$\rho_{TOT} = \rho_{free} + \rho_{bound}$$

Magneto*statics*:

$$\vec{F}_m = g_m \vec{B} - \frac{1}{c^2} g_m \vec{v} \times \vec{E}_{ext}$$

$$\vec{B}_g = \left(\frac{\mu_o}{4\pi}\right)\frac{g_m}{r^2}\hat{r}$$

$\vec{\nabla} \bullet \vec{B} = 0$ (always - no magnetic charges/monopoles)

$\Phi_m = \oint_S \vec{B} \bullet d\vec{a} = 0$ (n.b. closed surface S)

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J}_{TOT} = \mu_o \vec{J}_{free} + \mu_o \vec{J}_{bound}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \Phi_m^{encl} = \oint_C \vec{A} \bullet d\vec{\ell}$$

$$\vec{H} = \frac{1}{\mu}\vec{B} = \frac{1}{\mu_o}\vec{B} - \vec{\mathrm{M}}$$

$$\mu = \mu_o\left(1 + \chi_m\right), \quad K_m = \mu/\mu_o$$

$$\vec{\mathrm{M}} = \chi_m \vec{H}$$

$$\vec{\nabla} \times \vec{H} = -\vec{\nabla} \times \vec{\mathrm{M}}$$

$$\vec{J}_{bound} = \vec{\nabla} \times \vec{\mathrm{M}} \qquad \rho_m^{bound} = -\vec{\nabla} \bullet \vec{\mathrm{M}}$$

$$\vec{K}_{bound} = \vec{\mathrm{M}} \bullet \hat{n}\Big|_{\text{interface}} \qquad \sigma_m^{bound} = \vec{\mathrm{M}} \bullet \hat{n}\Big|_{\text{interface}}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_{free} \qquad \vec{\nabla} \times \vec{\mathrm{M}} = \vec{J}_{bound}$$

$$\frac{1}{\mu_o}\oint_C \vec{B} \bullet d\vec{\ell} = I_{TOT}^{encl}, \qquad \oint_C \vec{H} \bullet d\vec{\ell} = I_{free}^{encl},$$

$$\oint_C \vec{\mathrm{M}} \bullet d\vec{\ell} = I_{bound}^{encl}$$

$$\vec{J}_{bound} = \chi_m \vec{J}_{free}$$

$$\vec{\nabla} \bullet \vec{J}_{TOT} = -\frac{\partial \rho_{TOT}}{\partial t}$$

$$\vec{J}_{TOT} = \vec{J}_{free} + \vec{J}_{bound}$$

Electro*static* Boundary Conditions

$$E_2^{\parallel} = E_1^{\parallel} \Big|_{\text{interface}} \left(\left[E_2^{\parallel} - E_1^{\parallel} \right] \Big|_{\text{interface}} = 0 \right)$$

$$\left[D_2^{\parallel} - D_1^{\parallel} \right] \Big|_{\text{interface}} = \left[P_2^{\parallel} - P_1^{\parallel} \right] \Big|_{\text{interface}}$$

$$\varepsilon_o \left[E_2^{\perp} - E_1^{\perp} \right] \Big|_{\text{interface}} = \sigma_{TOT} \Big|_{\text{interface}} = \left(\sigma_{free} + \sigma_{bound} \right) \Big|_{\text{interface}}$$

$$\varepsilon_o \left(\frac{\partial V_2}{\partial n} \Big|_{\text{interface}} - \frac{\partial V_1}{\partial n} \Big|_{\text{interface}} \right) = -\sigma_{TOT} = -\left(\sigma_{free} + \sigma_{bound} \right) \Big|_{\text{interface}}$$

For linear dielectric media:

$$\varepsilon_2 \frac{\partial V_2}{\partial n} \Big|_{\text{interface}} - \varepsilon_1 \frac{\partial V_1}{\partial n} \Big|_{\text{interface}} = -\sigma_{free}$$

$$\left[P_2^{\perp} - P_1^{\perp} \right] \Big|_{\text{interface}} = -\sigma_{bound} \Big|_{\text{interface}}$$

$$\left[D_2^{\perp} - D_1^{\perp} \right] \Big|_{\text{interface}} = \sigma_{free} \Big|_{\text{interface}}$$

Magneto*static* Boundary Conditions

$$B_2^{\perp} = B_1^{\perp} \Big|_{\text{interface}} \left(\left[B_2^{\perp} - B_1^{\perp} \right] \Big|_{\text{interface}} = 0 \right)$$

$$\left[H_2^{\perp} - H_1^{\perp} \right] \Big|_{\text{interface}} = -\left[M_2^{\perp} - M_1^{\perp} \right] \Big|_{\text{interface}} = -\sigma_m^{pole} \Big|_{\text{interface}}$$

$$\frac{1}{\mu_o} \left[B_2^{\parallel} - B_1^{\parallel} \right] \Big|_{\text{interface}} = \vec{K}_{TOT} \times \hat{n} \Big|_{\text{interface}} = \left(\vec{K}_{free} + \vec{K}_{bound} \right) \times \hat{n} \Big|_{\text{interface}}$$

$$\frac{1}{\mu_o} \left(\frac{\partial \vec{A}_2}{\partial n} \Big|_{\text{interface}} - \frac{\partial \vec{A}_1}{\partial n} \Big|_{\text{interface}} \right) = -\vec{K}_{TOT} = -\left(\vec{K}_{free} + \vec{K}_{bound} \right)$$

For linear magnetic media:

$$\frac{1}{\mu_2} \frac{\partial \vec{A}_2}{\partial n} \Big|_{\text{interface}} - \frac{1}{\mu_1} \frac{\partial \vec{A}_1}{\partial n} \Big|_{\text{interface}} = -\vec{K}_{free}$$

$$\left[M_2^{\parallel} - M_1^{\parallel} \right] \Big|_{\text{interface}} = \vec{K}_{bound} \times \hat{n} \Big|_{\text{interface}}$$

$$\left[H_2^{\parallel} - H_1^{\parallel} \right] \Big|_{\text{interface}} = \vec{K}_{free} \times \hat{n} \Big|_{\text{interface}}$$